Name _____ Date _____

A Little Dash of Logic

Two Methods of Logical Reasoning

Vocabulary

Define each term in your own words.

1. inductive reasoning

2. deductive reasoning

Problem Set

Identify the specific information, the general information, and the conclusion for each problem situation.

1. You read an article in the paper that says a high-fat diet increases a person's risk of heart disease. You know your father has a lot of fat in his diet, so you worry that he is at higher risk of heart disease.

   Specific information: Your father has a lot of fat in his diet.

   General information: High-fat diets increase the risk of heart disease.

   Conclusion: Your father is at higher risk of heart disease.

2. You hear from your teacher that spending too much time in the sun without sunblock increases the risk of skin cancer. Your friend Susan spends as much time as she can outside working on her tan without sunscreen, so you tell her that she is increasing her risk of skin cancer when she is older.

3. Janice tells you that she has been to the mall three times in the past week, and every time there were a lot of people there. "It's always crowded at the mall," she says.

4. John returns from a trip out West and reports that it was over 100 degrees every day. "It's always hot out West," he says.

5. Mario watched 3 parades this summer. Each parade had a fire truck lead the parade. He concluded "A fire truck always leads a parade."

6. Ava read an article that said eating too much sugar can lead to tooth decay and cavities. Ava noticed that her little brother Phillip eats a lot of sugar. She concludes that Phillip's teeth will decay and develop cavities.

Determine whether inductive reasoning or deductive reasoning is used in each situation. Then determine whether the conclusion is correct and explain your reasoning.

7. Jason sees a line of 10 school buses and notices that each is yellow. He concludes that all school buses must be yellow.

   It is inductive reasoning because he has observed specific examples of a phenomenon—the color of school buses—and come up with a general rule based on those specific examples.

   The conclusion is not necessarily true. It may be the case, for example, that all or most of the school buses in this school district are yellow, while another school district may have orange school buses.

Name _____ Date _____

8. Caitlyn has been told that every taxi in New York City is yellow. When she sees a red car in New York City, she concludes that it cannot be a taxi.

9. Miriam has been told that lightning never strikes twice in the same place. During a lightning storm, she sees a tree struck by lightning and goes to stand next to it, convinced that it is the safest place to be.

10. Jose is shown the first six numbers of a series of numbers: 7, 11, 15, 19, 23, 27. He concludes that the general rule for the series of numbers is $a_n = 4n + 3$.

11. Isabella sees 5 red fire trucks. She concludes that all fire trucks are red.

12. Carlos is told that all garter snakes are not venomous. He sees a garter snake in his backyard and concludes that it is not venomous.

In each situation, identify whether each person is using inductive or deductive reasoning. Then compare and contrast the two types of reasoning.

13. When Madison babysat for the Johnsons for the first time, she was there 2 hours and was paid $30. The next time she was there for 5 hours and was paid $75. She decided that the Johnsons were paying her $15 per hour. The third time she went, she stayed for 4 hours. She tells her friend Jennifer that she makes $15 per hour babysitting. So, Jennifer predicted that Madison made $60 for her 4-hour babysitting job.

Madison used inductive reasoning to conclude that the Johnsons were paying her at a rate of $15 per hour. From that general rule, Jennifer used deductive reasoning to conclude that 4 hours of babysitting should result in a payment of $60. The inductive reasoning looks at evidence and creates a general rule from the evidence. By contrast, the deductive reasoning starts with a general rule and makes a prediction or deduction about what will happen in a particular instance.

14. When Holly was young, the only birds she ever saw were black crows. So, she told her little brother Walter that all birds are black. When Walter saw a bluebird for the first time, he was sure it had to be something other than a bird.

15. Tamika is flipping a coin and recording the results. She records the following results: heads, tails, heads, tails, heads, tails, heads. She tells her friend Javon that the coin alternates between heads and tails for each toss. Javon tells her that the next time the coin is flipped, it will definitely be tails.

Name _____ Date _____

16. John likes to watch the long coal trains moving past his house. Over the weeks of watching he notices that every train going east is filled with coal, but the trains heading west are all empty. He tells his friend Richard that all trains heading east have coal and all trains heading west are empty. When Richard hears a train coming from the west, he concludes that it will certainly be filled with coal.

17. Vance earned $60 mowing 5 lawns last weekend for the Greenvalley Homeowners Association. Vance concluded that he earned $12 for each lawn. Vance told Sherwin that he planned to mow 7 lawns for Greenvalley next weekend. Sherwin concluded that Vance would earn $84 mowing the 7 lawns.

18. As a child, the only frogs Emily ever saw were green. Emily told Juan that all frogs are green. When Juan visited a zoo and saw a blue poison dart frog he concluded that it must be something other than a frog.

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What's Your Conclusion?

Understanding Conditional Statements, Arguments, and Truth Tables

Vocabulary

Choose the term or terms from the box that best completes each statement.

conditional statement	propositional form	hypothesis
propositional variables	conclusion	truth value
truth table		

1. The _____ of a conditional statement is the variable q.

2. A _____ is a statement that can be written in the form "If p, then q." This form is also known as the _____.

3. A _____ is a table that summarizes all possible truth values for a conditional statement $p \rightarrow q$.

4. The _____ of a condition statement is the variable p.

5. In a conditional statement, the variables p and q are _____.

6. The _____ of a conditional statement is whether the statement is true or false.

Problem Set

Read each conditional statement and conclusion. Then write the additional statement required to reach the conclusion.

1. Conditional Statement: If my age is 15 now, then I will be 16 on my next birthday.
   Statement: I am not 15 now.
   Conclusion: Therefore, I will not be 16 on my next birthday.

2. Conditional Statement: If it rains today, then I will need to take my umbrella.
   Statement:
   Conclusion: Therefore, I need to take my umbrella.

3. Conditional Statement: If you had read the notice, then you would have known there was no class today.
   Statement:
   Conclusion: Therefore, I knew there was no class today.

4. Conditional Statement: If I had studied more, then I could have gotten an A on my logic test.
   Statement:
   Conclusion: Therefore, I did not get an A on my logic test.

5. Conditional Statement: If the sun is shining today, then I will wear my sunglasses.
   Statement:
   Conclusion: Therefore, I am not wearing my sunglasses.

6. Conditional Statement: If it snows today, then I will go skiing.
   Statement:
   Conclusion: Therefore, I am going skiing.

Name _____ Date _____

For each conditional statement, draw a solid line beneath the hypothesis. Then draw a dotted line beneath the conclusion.

7. If it is sunny tomorrow, we will go to the beach.

8. If the groundhog sees its shadow, there will be six more weeks of winter.

9. If a and b are real numbers, then $a^2 + b^2$ is greater than or equal to 0.

10. If I am smiling, then I am happy.

11. If I get a raise, then I will buy a new car.

12. If I pass my final exam, then I will pass my finance course.

Complete the truth table for each conditional statement. Then explain what each row means in the truth table.

13. "If I can play the violin, then I can join the orchestra."

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Row 1: If p is true, then I can play the violin. If q is true, then I can join the orchestra. It is true that if I can play the violin, I can join the orchestra, so the truth value of the conditional statement is true.

Row 2: If p is true, then I can play the violin. If q is false, then I cannot join the orchestra. It is false that if I can play the violin, I cannot join the orchestra, so the truth value of the conditional statement is false.

Row 3: If p is false, then I cannot play the violin. If q is true, then I can join the orchestra. It could be true that if I cannot play the violin, I can join the orchestra, so the truth value of the conditional statement in this case is true. (For instance, if I play another instrument.)

Row 4: If p is false, then I cannot play the violin. If q is false, then I cannot join the orchestra. It could be true that if I cannot play the violin, I cannot join the orchestra, so the truth value of the conditional statement in this case is true.

14. "If $n = 2$, then $n^2 = 4$."

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

15. "If a plant is an oak, then the plant is a tree."

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

Name _____ Date _____

16. "If your mode of transportation is a motorcycle, then your mode of transportation has two wheels."

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

17. If the traffic light is red, then the car is stopped.

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

18. If the electricity is on, then the light is lit.

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

Name _____ Date _____

Write the converse of each conditional statement.

19. If today is Tuesday, then Janis has a piano lesson after school.
If Janis has a piano lesson after school, then today is Tuesday.

20. If that animal is a dog, then it has four legs.

21. If he believed that the sky is green, then he would be crazy.

22. If one book costs $10, then five books cost $50.

23. If the flower if red, then it is a rose.

24. If Jamal is sleeping, then it is nighttime.

Write the inverse of each conditional statement.

25. If you go to the grocery store on Saturday, then there will be very long lines.
If you do not go to the grocery store on Saturday, then there will not be very long lines.

26. If Krista gets an A on her history test, then she is allowed to spend the weekend with her friend.

27. If the bus does not arrive on time, then Milo will be late for work.

28. If there is a chance of rain this weekend, then Liza will cancel her camping trip.

29. If the figure has 3 sides, then the figure is a triangle.

30. If a number is less than 0, then it is negative.

Write the contrapositive of each conditional statement.

31. If a triangle is an equilateral triangle, then all of its sides are equal.
If the sides of a triangle are not all equal, then the triangle is not an equilateral triangle.

32. If it is dark outside, then it is nighttime.

33. If there are more than 30 students in this classroom, then it is too crowded.

34. If the next animal you see is a kangaroo, then you are in Australia.

35. If a figure has 10 sides, then the figure is a decagon.

36. If a number is greater than zero, then the number is positive.

For each conditional statement, write the converse of that statement. If possible, write a true biconditional statement. If not possible, explain why.

37. If N is divisible by 10, then the last digit in N is 0.
If the last digit in N is 0, then N is divisible by 10. True.
Biconditional statement: N is divisible by 10 if and only if the last digit in N is 0.

38. If two triangles are congruent, then the triangles have equal angles.

Name _____ Date _____

39. If the last digit in N is 5, then N is divisible by 5.

40. If x is greater than 0, then x^3 is greater than 0.

41. If a triangle has no equal sides, then the triangle is scalene.

42. If a triangle has an angle more than 90°, then the triangle is obtuse.

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Name _____ Date _____

Proofs Aren't Just for Geometry

Introduction to Direct and Indirect Proof with the Properties of Numbers

Vocabulary

Define the term in your own words.

1. proof by contradiction

Problem Set

Identify whether the commutative property, associative property, identity property, inverse property of addition, or inverse property of multiplication explains why each statement is true.

1. $(5 + 3) + 4 = 5 + (3 + 4)$

Associative property of addition

2. $172.3 + (-172.3) = 0$

3. $107 \cdot \frac{1}{107} = 1$

4. $12 \cdot 23 = 23 \cdot 12$

5. $13{,}416.7 \cdot 1 = 13{,}416.7$

6. $37 + 92 = 92 + 37$

7. $13(24 \cdot 117) = (13 \cdot 24)117$

8. $16\frac{3}{5} + 0 = 16\frac{3}{5}$

Use the distributive property to calculate each value.

9. $12(6 + 10)$

$12(6 + 10) = 12(6) + 12(10) = 72 + 120 = 192$

10. $(13 + 22) \cdot 4$

11. $4(x + y)$

12. $13(a - b)$

13. $mn - mp$

14. $4d + 4e$

Prove or disprove each statement.

15. If $a(b + c) = b(a + c) + ac$, then either $b = 0$ or $c = 0$ (or both).

$a(b + c) = b(a + c) + ac$	
$ab + ac = ba + bc + ac$	Distributive property
$ab + ac = ab + bc + ac$	Commutative property of multiplication
$ab + ac - ac = ab + bc + ac - ac$	Subtraction law of equality
$ab = ab + bc$	Inverse property of addition
$ab - ab = ab + bc - ab$	Subtraction property of equality
$0 = bc + ab - ab$	Commutative property
$0 = bc$	Additive inverse
$b = 0$ or $c = 0$ (or both)	If a product is equal to zero, at least 1 factor in the product is equal to zero.

Name _____ Date _____

16. If $ab + bc + ac = a(b + c)$, then either $b = 0$ or $c = 0$ (or both).

17. If $(x + a)(x + b) = x^2 + ab$ for all x, then either $a = 0$ or $b = 0$.

18. If $(a + b)c = c(a - b)$, then either $b = 0$ or $c = 0$.

19. If a and b are real numbers, then $a(b + 2) = ab + 2$.

16

20. If a, b, and c are real numbers, then $\frac{a}{b + c} = \frac{a}{b} + \frac{a}{c}$.

Name _____ Date _____

Your Oldest Likes Spinach?

Using Logic to Solve Problems, Part 1

Problem Set

Solve each problem.

1. When Tia asked her neighbor Shelia her birth date, she answered her with this riddle: The sum of the month, day, and the last two digits of the year is 87. The day is four times the month and is the last two digits of the year divided by 6. What is the date of Shelia's birthday?

   Day = 4 × *month*, so *day* must be a multiple of 4.

   Days of the month that are multiples of 4 are: 4, 8, 12, 16, 20, 24, and 28.

   $Day = \frac{year}{6}$, or $year = day \times 6$

   Multiply each possible day by 6: 4 × 6 = 24, 8 × 6 = 48, 12 × 6 = 72, and 16 × 6 = 96
   (Stop there because further calculations produce a three-digit number.)

   So, the day is 4, 8, 12, or 16, and the year is 1924, 1948, 1972, or 1996.

   Because the sum of *day*, *month*, and *the last 2 digits of the year* is 87, you can eliminate 1996 as the year (and thus 16 as the day). Going back to *day* = 4 × *month*, the month is 1, 2, or 3.

   Try the combinations: 4 + 1 + 24 ≠ 87, 8 + 2 + 48 ≠ 87, and 12 + 3 + 72 = 87

   So, Shelia's birthday is March 12, 1972.

2. Suzanne is 3 years younger than Chloe. Theodore is older than Chloe. The sum of Theodore and Suzanne's ages is 11. Theodore and Chloe are both in elementary school. Chloe is closer in age to Theodore than to Suzanne. How old is Chloe?

3. Franklin has 50¢ in his pocket. There are less than 10 coins in his pocket. There are three different types of coins in his pocket. The number of the smallest-sized coins is the same as the number of the largest-sized coins. There is more than one of the least-value coin. What coins are in Franklin's pocket?

4. Eva, Mei, and Grace each did something different on Saturday (movies, party, or bowling). Assume only one of the following statements is true. Determine who did what activity.

A. Grace went to the movies.

B. Grace did not go to a party.

C. Mei did not go to a party.

D. Mei did not go bowling.

5. James was helping his grandmother hang three framed photos on the wall. She gave him the following directions. Hang the black frame to the left of the silver frame. Hang the flower photo to the right of the bird photo. Hang the tree photo to the left of the brown frame. Hang the brown frame to the left of the silver frame. Which photo is in which frame and in what order are they from left to right?

Name _____ Date _____

6. Rita lives on Miller Street. There are five houses on her side of the street. The house numbers on her side of the street are consecutive odd numbers. Angelo's house number is the highest number on Rita's side. The sum of the house numbers is 545. What is Angelo's address?

Complete the logic puzzle grid to solve each problem.

7. Marcus signed up for Math Club, Yearbook, and Jazz Band this year. He asked his counselor where each after-school activity meets. His counselor gave him the following instructions:

Mr. Juarez meets his group in room 10. Yearbook meets in room 6. Mr. Dalton advises either Yearbook or Jazz band. Jazz is not advised by Mrs. Aiello, but it meets in room 9.

Complete the grid and then list each of Marcus's after-school activities, their advisors, and their room numbers.

		Advisor			Activity		
		Mr. Juarez	Mr. Dalton	Mrs. Aiello	Math Club	Yearbook	Jazz Band
Room	Room 6	X	X	O	X	O	X
	Room 9	X	O	X	X	X	O
	Room 10	O	X	X	O	X	X
Activity	Math Club	O	X	X			
	Yearbook	X	X	O			
	Jazz Band	X	O	X			

Math Club is advised by Mr. Juarez and meets in Room 10.
Yearbook is advised by Mrs. Aiello and meets in Room 6.
Jazz Band is advised by Mr. Dalton and meets in Room 9.

8. The Student Council is organizing information about the Fall Carnival Fundraiser. Three booths are planned: a basketball shoot, a dunking booth, and a balloon pop. Each booth has two, three, or four volunteers and key rings, homework passes, or glow sticks for prizes. The basketball shoot has glow sticks for prizes. The dunking booth has more than two volunteers and does not have key rings for prizes. The booth with three volunteers has homework passes for prizes. The balloon pop has fewer volunteers than the basketball shoot.

Complete the grid and then list each booth with the correct number of volunteers and prize offered.

		Prizes: Key rings	Prizes: Homework passes	Prizes: Glow sticks	Booths: Basketball shoot	Booths: Dunking booth	Booths: Balloon pop
Volunteers	Two						
	Three						
	Four						
Booths	Basketball shoot						
	Dunking booth						
	Balloon pop						

Name _____ Date _____

9. Hector, Ella, and Mitsu placed first, second, and third in the Science Fair. Ella's project won first place. Hector's project was neither in Physics nor in third place. The person who placed third did a Chemistry experiment.

Complete the grid and then list each student with the type of science and the place their project won.

		Science			Student		
		Biology	Chemistry	Physics	Ella	Hector	Mitsu
Place	1						
	2						
	3						
Student	Ella						
	Hector						
	Mitsu						

10. All of the trophies in the school's trophy case were removed for cleaning. The secretary must use the following instructions to put them back on the proper shelves. The case has three shelves each holding trophies from a different sport: football, basketball, and field hockey. Trophies from each sport are different types: statues, plaques, and cups. The bottom shelf does not display field hockey trophies. The middle shelf displays plaques. Basketball trophies are statues. The top shelf does not display football trophies or cups.

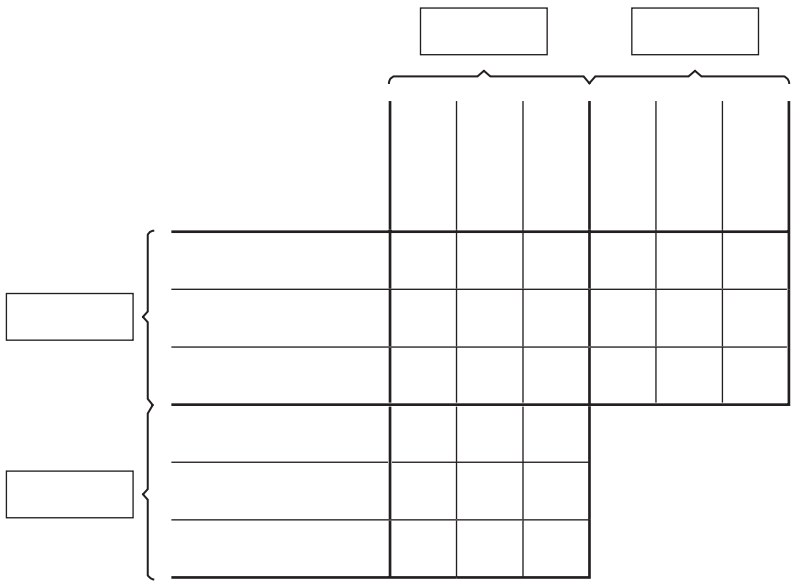
Complete the grid and then list each shelf with the correct sport and type of trophy.

		Trophy			Sport		
		Statue	Plaque	Cup	Football	Basketball	Field hockey
Shelf	Top						
	Middle						
	Bottom						
Sport	Football						
	Basketball						
	Field hockey						

Name _____ Date _____

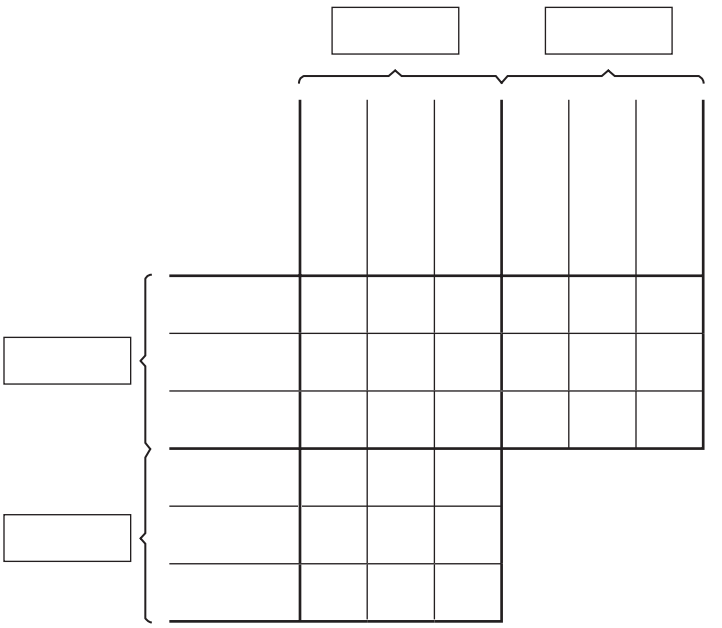
11. John, Yasmine, and Zach each have a different type of bag: backpack, messenger bag, or duffle bag. Each bag is made of a different type of material: leather, canvas, or nylon. The backpack is leather. John's bag is not canvas or leather. Yasmine's bag is not a duffle bag or leather.

Label and complete the grid and then list each student with the type and material of his or her bag.



12. There are three middle schools in Clayton County: Carver, Wiley, and Taft. Each school has a different school mascot (eagle, bulldog, or hawk) and school colors (orange/red, red/black, or black/gold). The hawk's colors do not include black and Wiley's colors do not include red. Taft does not have a bird as its mascot.

Label and complete the grid and then list each school, its mascot, and its colors.



Name _____ Date _____

Shoes and Math Scores?

Using Logic to Solve Problems, Part 2

Problem Set

Complete the grid to solve each problem.

1. Farmer Gray, Farmer White, Farmer Brown, and Farmer Green each grow a different crop (wheat, cotton, soy, or corn) and each has a different number of acres (250, 400, 500, and 750). Complete the grid and then list each farmer, his crop, and the number of acres on his farm.

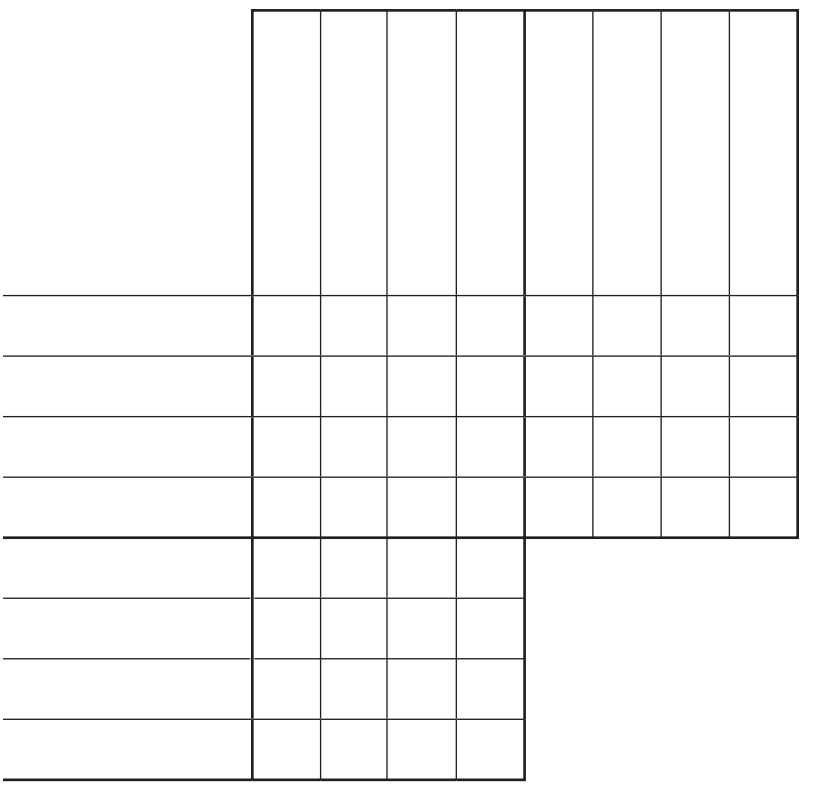
   - Farmer Brown grows corn.
   - The farmer who grows soy has the largest farm.
   - Farmer Green and Farmer White do not grow crops that start with a "c."
   - Farmer Green is allergic to soy and does not have the smallest farm.
   - There are 500 acres of cotton.

	Wheat	Cotton	Soy	Corn	250 acres	400 acres	500 acres	750 acres
Farmer Gray	X	O	X	X	X	X	O	X
Farmer White	X	X	O	X	X	X	X	O
Farmer Brown	X	X	X	O	O	X	X	X
Farmer Green	O	X	X	X	X	O	X	X
250 acres	X	X	X	O				
400 acres	O	X	X	X				
500 acres	X	O	X	X				
750 acres	X	X	O	X				

Farmer Gray grows cotton on 500 acres.
Farmer White grows soy on 750 acres.
Farmer Brown grows corn on 250 acres.
Farmer Green grows wheat on 400 acres.

2. Noah, J.J., Demari, and Victor each go to and from school in different ways. The possible ways are: walking, riding the bus, riding in a car, or biking. Complete the grid and then list each student, his method for getting to school, and his method for getting home from school.

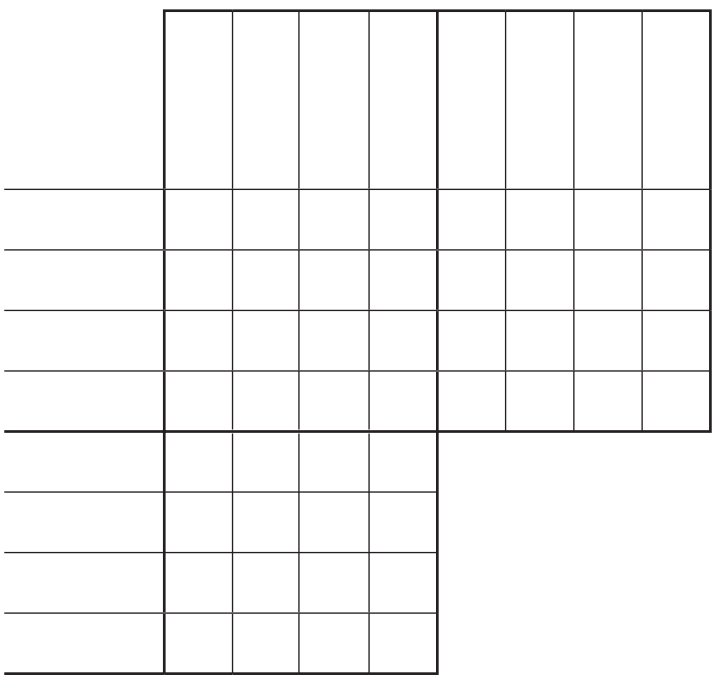
- The boy who bikes to school also bikes home.
- Victor walks to school but is too tired in the afternoon to walk home.
- Noah and Demari both ride the bus.
- J.J. does not ride in a car.
- Noah's dad drops him off at school on his way to work.



Name _____ Date _____

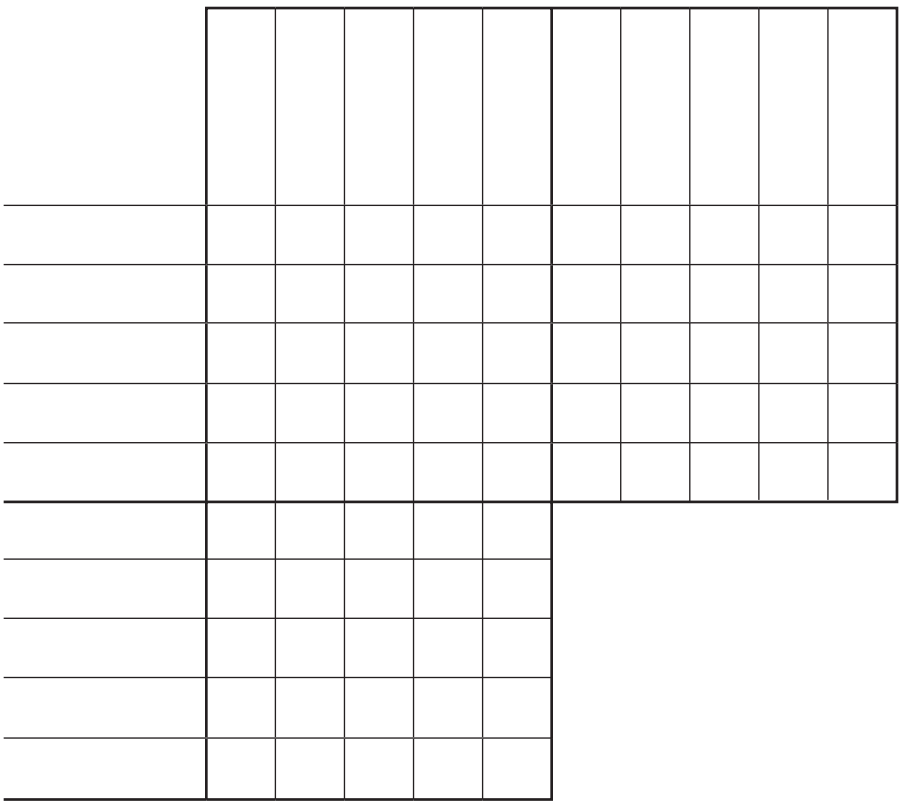
3. There are four types of recycling bins at the school. Each is for a different material: aluminum, paper, glass, or plastic. Each is a different color: yellow, blue, green, or gray. Each bin is also a different shape: round, octagonal, rectangular, or square. Complete the grid and then list the color and shape of each bin and the type of material it holds.

- The bin for plastic is not round or octagonal.
- The green bin is for glass.
- Either paper or glass is collected in the square bin.
- The rectangular bin is yellow.
- Kyle threw a soda can in the round, blue bin.
- The square bin is not green.



4. Angelo, Rashan, Vance, Chen, and Adam ran the 800-meter run. They finished first, second, third, fourth, and fifth. Each participant received a different colored ribbon (red, gold, blue, purple, and white) for his achievement. Complete the grid and then list each runner, the place he finished, and the color of his ribbon.

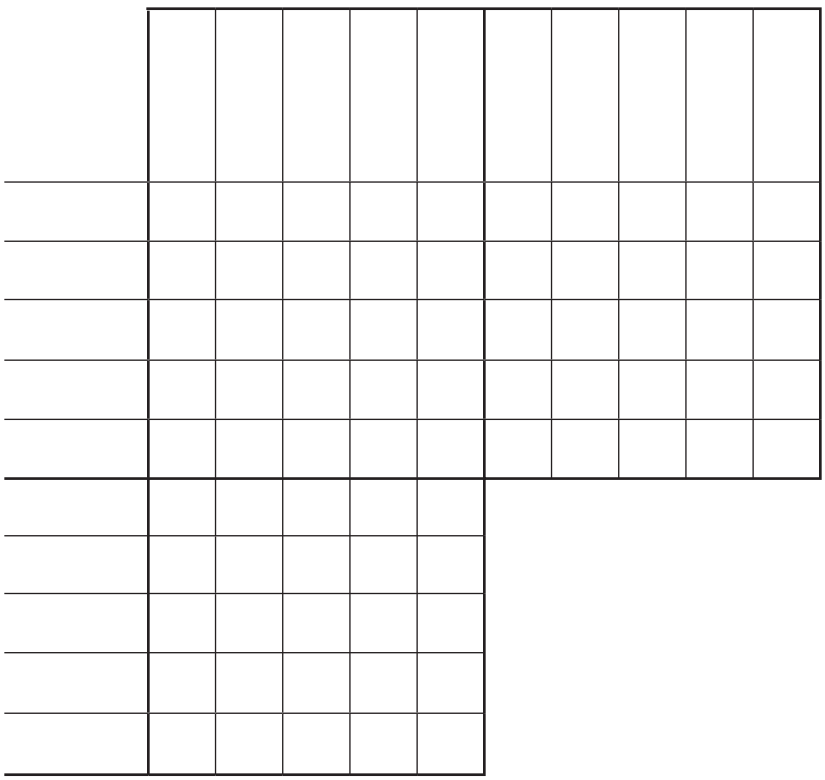
- The blue and white ribbons are for first and last place (not necessarily in that order).
- Adam was not first or last.
- Angelo finished fourth.
- Chen got the purple ribbon and finished just before Angelo.
- Rashan broke the school record.
- The red, purple, and white ribbons were given to the top three runners (not necessarily in that order).



Name _____ Date _____

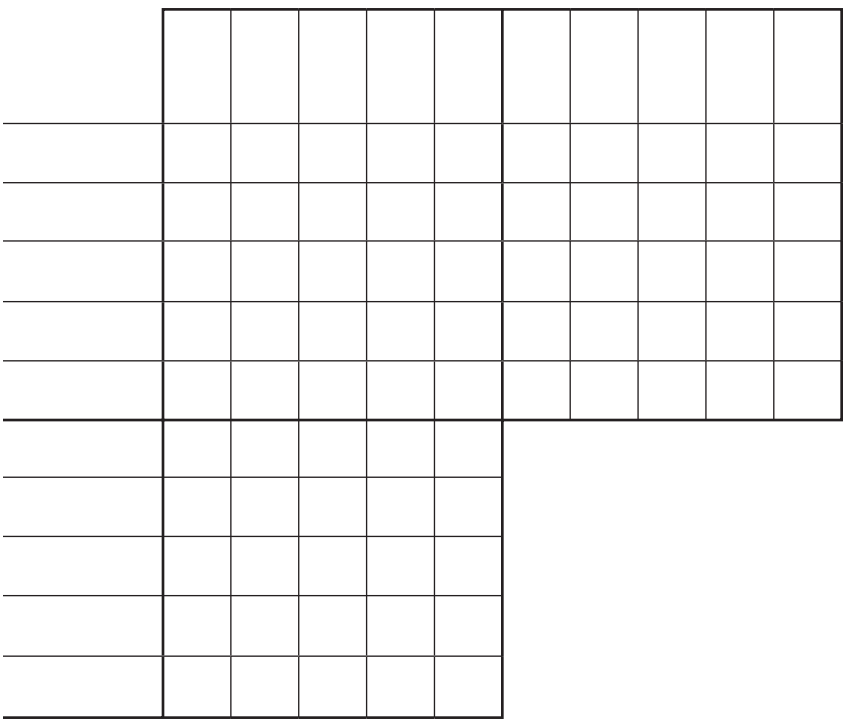
5. The Bryants, the Levines, the Smiths, the Fogartys, and the Singhs all live on the same street in houses 425, 427, 429, 431, and 433. Each of their houses is a different style: brick, Tudor, modern, Victorian, or farmhouse. Complete the grid and then list each family, their house style, and their house number.

- The Fogartys live in the Victorian at the end of the block.
- The Singhs do not live in the brick house or the farmhouse.
- The house at 431 is a modern house.
- The Levines live in the brick house between the Singhs and the Bryants.
- The modern house is between the Victorian and the Tudor.



6. There are five dogs at the shelter: Jack, Lady, Sadie, Butch, and Max. Each dog is a different size: extra large, large, medium, small, or toy. Each dog is a different color: black, brown, yellow, gray, or white. Complete the grid and then list each dog, its size, and its fur color.

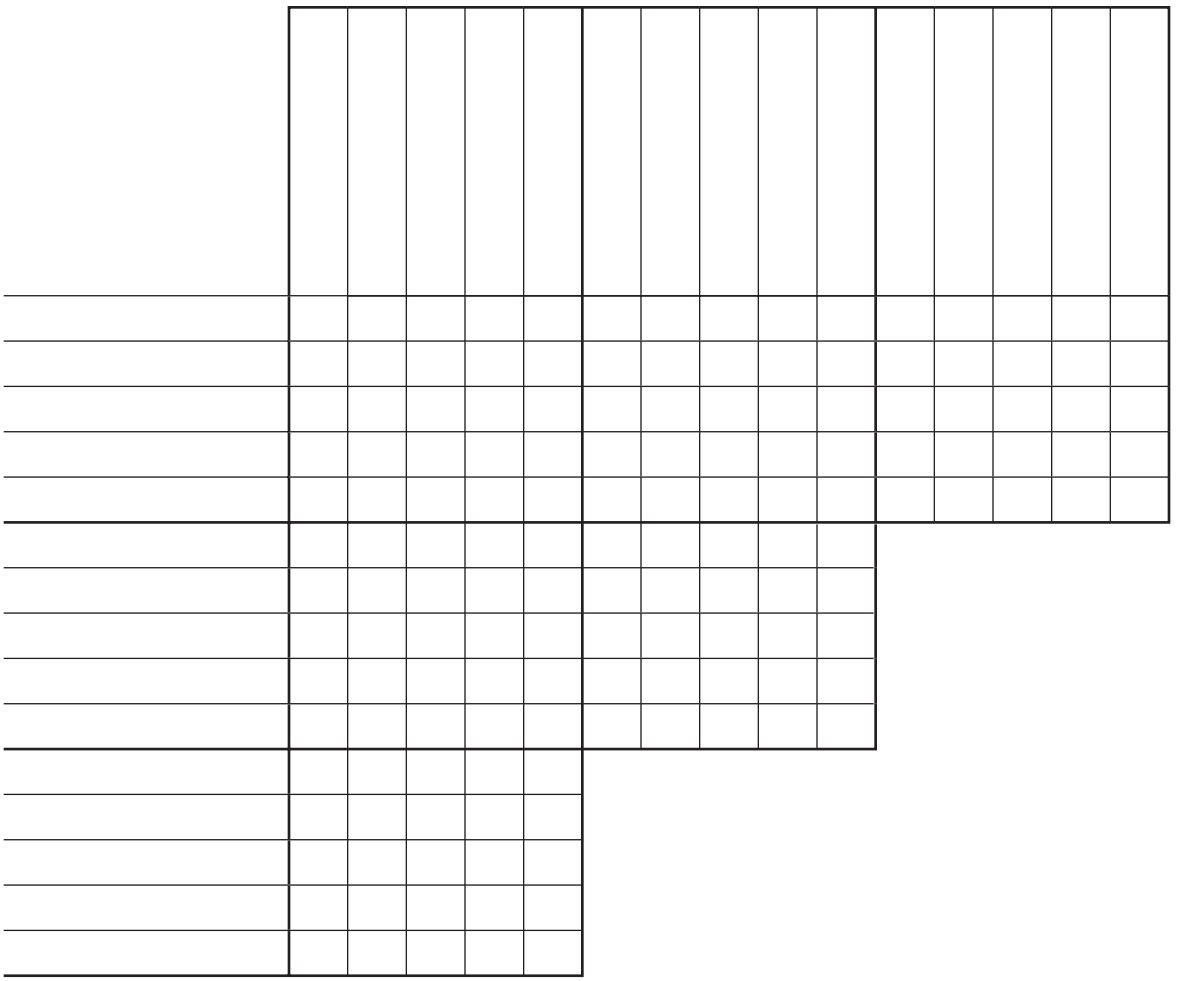
- The largest dog is not yellow, gray, or black.
- Jack is a small dog.
- Neither Butch nor Lady is the toy dog.
- Sadie is a big, white dog.
- A boy wanted the large, yellow dog, but wanted a girl dog, so he adopted the smaller, brown dog.
- Max is not black.



Name _____ Date _____

7. Parker polled her friends Ty, Maddie, Eva, Ben, and Zoe. She found that their favorite school subjects are math, art, science, drama, and history. Their favorite lunches are pizza, salad, tacos, turkey sub, and spaghetti. Their favorite activities are reading, riding a bike, walking the dog, playing on the computer, and listening to music. Complete the grid and then list each friend and their favorite subject, lunch, and activity.

- Eva loves spaghetti.
- Ben does not have a dog.
- Maddie brings her sketchbook with her on her bike rides in the park.
- Zoe is a vegetarian and an animal lover.
- Ben likes math and science.
- The person who likes science spends a lot of time on the computer.
- Eva does not like acting, but she likes to read plays.
- Ty likes pizza, but Maddie prefers Mexican food.
- The person who likes music also likes history.



Name _____ Date _____

8. In Chambersville there are five parks (Spring, Center, Hill, Miller, and North). Each park has a unique sports area, number of picnic tables available, and special feature. The sports areas are a tennis court, volleyball court, baseball field, bike path, or basketball court. The numbers of picnic tables available are 2, 4, 8, 10, or 20. The special features are a pond, merry-go-round, pool, gazebo, and amphitheatre. Complete the grid and list each park name with its sports area, number of picnic tables, and special feature.

- You cannot play a team sport at Hill Park.
- Center Park has the most picnic tables.
- The amphitheatre is in the park with the basketball court.
- The park with a tennis court does not have a special feature with water.
- Fans attending the baseball games can ride the merry-go-round.
- North Park has half the number of picnic tables as Center Park, but it has plenty of seating in the amphitheatre.
- At one park you can feed the ducks from your bike, but there are only 2 tables for picnicking.
- There are 4 picnic tables around the gazebo.
- Swimming is not allowed at Hill Park, but Spring Park is open for swimming all summer.

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