Graphs, Equations, and Inequalities

2

2.1

The Plane!

Modeling Linear Situations

LEARNING GOALS

In this lesson, you will:

- Complete tables and graphs, and write equations to model linear situations.
- Analyze multiple representations of linear relationships.
- Identify units of measure associated with linear relationships.
- Determine solutions both graphically and algebraically.
- Determine solutions to linear functions using intersection points.

KEY TERMS

- first differences
- solution
- intersection point

"Ladies and gentlemen, at this time we ask that all cell phones and pagers be turned off for the duration of the flight. All other electronic devices must be turned off until the aircraft reaches 10,000 feet. We will notify you when it is safe to use such devices."

Flight attendants routinely make announcements like this on airplanes shortly before takeoff and landing. But what's so special about 10,000 feet?

When a commercial airplane is at or below 10,000 feet, it is commonly known as a "critical phase" of flight. This is because research has shown that most accidents happen during this phase of the flight—either takeoff or landing. During critical phases of flight, the pilots and crew members are not allowed to perform any duties that are not absolutely essential to operating the airplane safely.

And it is still not known how much interference cell phones cause to a plane's instruments. So, to play it safe, crews will ask you to turn them off.

PROBLEM 1 Analyzing Tables

A 747 airliner has an initial climb rate of 1800 feet per minute until it reaches a height of 10,000 feet.

1. Identify the independent and dependent quantities in this problem situation. Explain your reasoning.

2

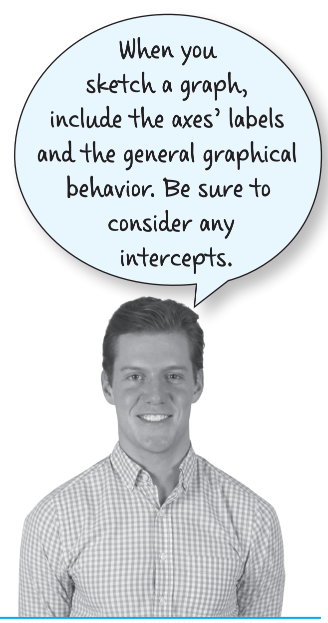
2. Describe the units of measure for:

a. the independent quantity (the input values).

b. the dependent quantity (the output values).

3. Which function family do you think best represents this situation? Explain your reasoning.

4. Draw and label two axes with the independent and dependent quantities and their units of measure. Then sketch a simple graph of the function represented by the situation.



5. Write the independent and dependent quantities and their units of measure in the table. Then, calculate the dependent quantity values for each of the independent quantity values given.

	Independent Quantity	Dependent Quantity
Quantity		
Units		
	0	
	1	
	2	
	2.5	
	3	
	3.5	
	5	
Expression	t	

6. Write an expression in the last row of the table to represent the dependent quantity. Explain how you determined the expression.

2

Let's examine the table to determine the unit rate of change for this situation. One way to determine the unit rate of change is to calculate *first differences*. Recall that **first differences** are determined by calculating the difference between successive points.

7. Determine the first differences in the section of the table shown.

Time (minutes)	Height (feet)
0	0
1	1800
2	3600
3	5400

$1 - 0 = 1$

$2 - 1 = 1$

$3 - 2 = 1$

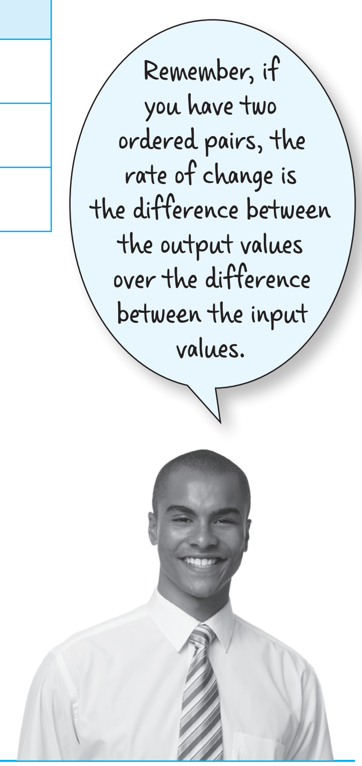
First Differences

8. What do you notice about the first differences in the table? Explain what this means.

Another way to determine the unit rate of change is to calculate the rate of change between any two ordered pairs and then write each rate with a denominator of 1.

9. Calculate the rate of change between the points represented by the given ordered pairs in the section of the table shown. Show your work.

Time (minutes)	Height (feet)
2.5	4500
3	5400
5	9000



a. (2.5, 4500) and (3, 5400)

b. (3, 5400) and (5, 9000)

c. (2.5, 4500) and (5, 9000)

10. What do you notice about the rates of change?

11. Use your answers from Question 7 through Question 10 to describe the difference between a rate of change and a unit rate of change.

12. How do the first differences and the rates of change between ordered pairs demonstrate that the situation represents a linear function? Explain your reasoning.

13. Alita says that in order for a car to keep up with the plane on the ground, it would have to travel at only 20.5 miles per hour. Is Alita correct? Why or why not?

PROBLEM 2 Analyzing Equations and Graphs

1. Complete the table shown for the problem situation described in Problem 1, *Analyzing Tables*. First, determine the unit of measure for each expression. Then, describe the contextual meaning of each part of the function. Finally, choose a term from the word box to describe the mathematical meaning of each part of the function.

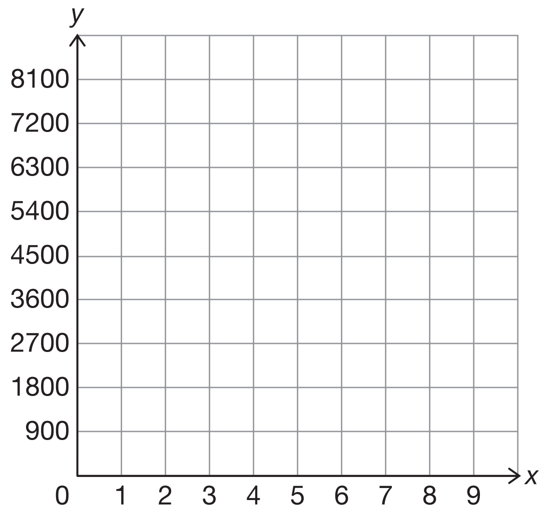
output value	input value	rate of change

		What It Means	
Expression	**Unit**	**Contextual Meaning**	**Mathematical Meaning**
t			
1800			
$1800t$			

2. Write a function, $h(t)$, to describe the plane's height over time, t.

3. Which function family does $h(t)$ belong to? Is this what you predicted back in Problem 1, Question 3?

4. Use your table and function to create a graph to represent the change in the plane's height as a function of time. Be sure to label your axes with the correct units of measure and write the function.



a. What is the slope of this graph? Explain how you know.

b. What is the *x*-intercept of this graph? What is the *y*-intercept? Explain how you determined each intercept.

c. What do the *x*- and *y*-intercepts mean in terms of this problem situation?

Let's consider how to determine the height of the plane, given a time in minutes, using function notation.

To determine the height of the plane at 2 minutes using your function, substitute 2 for t every time you see it. Then, simplify the function.

$$h(t) = 1800t$$

Substitute 2 for t. ⟶ $h(2) = 1800(2)$

$$h(2) = 3600$$

Two minutes after takeoff, the plane is at 3600 feet.

5. List the different ways the height of the plane is represented in the example.

6. Use your function to determine the height of the plane at each given time in minutes. Write a complete sentence to interpret your solution in terms of the problem situation.

a. $h(3) =$ ____________

b. $h(3.75) =$ ____________

c. $h(5.1) =$ ____________

d. $h(-4) =$ ____________

PROBLEM 3 Connecting Approaches

Now let's consider how to determine the number of minutes the plane has been flying (the input value) given a height in feet (the output value) using function notation.

To determine the number of minutes it takes the plane to reach 7200 feet using your function, substitute 7200 for $h(t)$ and solve.

$$h(t) = 1800t$$

Substitute 7200 for $h(t)$. →

$$7200 = 1800t$$

$$\frac{7200}{1800} = \frac{1800t}{1800}$$

$$4 = t$$

After takeoff, it takes the plane 4 minutes to reach a height of 7200 feet.

1. Why can you substitute 7200 for $h(t)$?

2. Use your function to determine the time it will take the plane to reach each given height in feet. Write a complete sentence to interpret your solution in terms of the problem situation.

a. 5400 feet

b. 9000 feet

c. 3150 feet

d. 4500 feet

You can also use the graph to determine the number of minutes the plane has been flying (input value) given a height in feet (output value). Remember, the **solution** of a linear equation is any value that makes the open sentence true. If you are given a graph of a function, a solution is any point on that graph. The graph of any function, f, is the graph of the equation $y = f(x)$. If you have intersecting graphs, a solution is the ordered pair that satisfies both functions at the same time, or the **intersection point** of the graphs.

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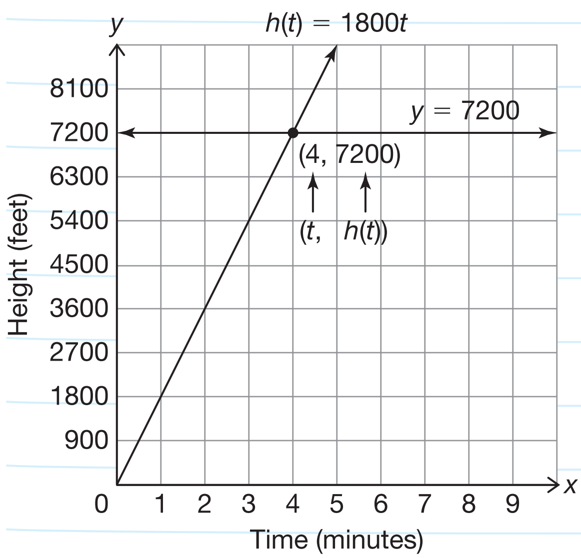
To determine how many minutes it takes for the plane to reach 7200 feet using your graph, you need to determine the intersection of the two graphs represented by the equation $7200 = 1800t$.

First, graph each side of the equation and then determine the intersection point of the two graphs.



$h(t) = 1800t$

$7200 = 1800t$

$\downarrow \qquad \downarrow$

$y = 7200 \qquad y = 1800x$

Solution: (4, 7200)

After takeoff, it takes the plane 4 minutes to reach a height of 7200 feet.

3. What does $(t, h(t))$ represent?

4. Explain the connection between the form of the function $h(t) = 1800t$ and the equation $y = 1800x$ in terms of the independent and dependent quantities.

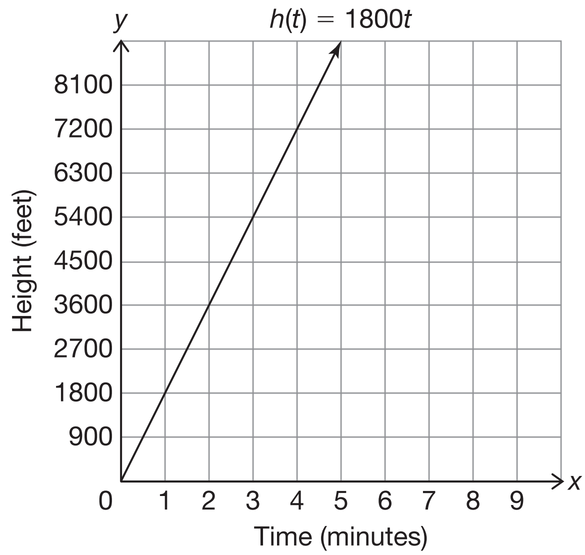
5. Use the graph to determine how many minutes it will take the plane to reach each height.

a. $h(t) = 5400$

b. $h(t) = 9000$

c. $h(t) = 3150$

d. $h(t) = 4500$



6. Compare and contrast your solutions using the graphing method to the solutions in Question 2, parts (a) through (d) where you used an algebraic method. What do you notice?

Talk the Talk

You just worked with different representations of a linear function.

1. Describe how a linear function is represented:

   a. in a table.

   b. in a graph.

   c. in an equation.

2. Name some advantages and disadvantages of the graphing method and the algebraic method when determining solutions for linear functions.

3. Do you think the graphing method for determining solutions will work for any function?

You solved several linear equations in this lesson. Remember, the Addition, Subtraction, Multiplication, and Division Properties of Equality allow you to balance and solve equations. The Distributive Property allows you to rewrite expressions to remove parentheses, and the Commutative and Associative Properties allow you to rearrange and regroup expressions.

4. Solve each equation and justify your reasoning.

a. $7x + 2 = -12$

b. $4(x + -7) + 12 = 20$

c. $14x - 13 = 9x + 1$

d. $\frac{x + 2}{6} = \frac{2}{5}$

Be prepared to share your solutions and methods.

2

2

2.2

What Goes Up Must Come Down

Analyzing Linear Functions

LEARNING GOALS

In this lesson, you will:

- Complete tables and graphs, and write equations to model linear situations.
- Analyze multiple representations of linear relationships.
- Identify units of measure associated with linear relationships.
- Determine solutions to linear functions using intersection points and properties of equality.
- Determine solutions using tables, graphs, and functions.
- Compare and contrast different problem-solving methods.
- Estimate solutions to linear functions.
- Use a graphing calculator to analyze functions and their graphs.

The dollar is just one example of currency used around the world. For example, Swedes use the krona, Cubans use the peso, and the Japanese use the yen. This means that if you travel to another country you will most likely need to exchange your U.S. dollars for a different currency. The exchange rate represents the value of one country's currency in terms of another—and it is changing all the time. In some countries, the U.S. dollar is worth more. In other countries, the dollar is not worth as much.

Why would knowing the currency of another country and the exchange rate be important when planning trips?

PROBLEM 1 As We Make Our Final Descent

At 36,000 feet, the crew aboard the 747 airplane begins making preparations to land. The plane descends at a rate of 1500 feet per minute until it lands.

1. Compare this problem situation to the problem situation in Lesson 2.1, *The Plane!* How are the situations the same? How are they different?

2. Complete the table to represent this problem situation.

	Independent Quantity	Dependent Quantity
Quantity		
Units		
	0	
	2	
	4	
	6	
		18,000
		6000
Expression	t	

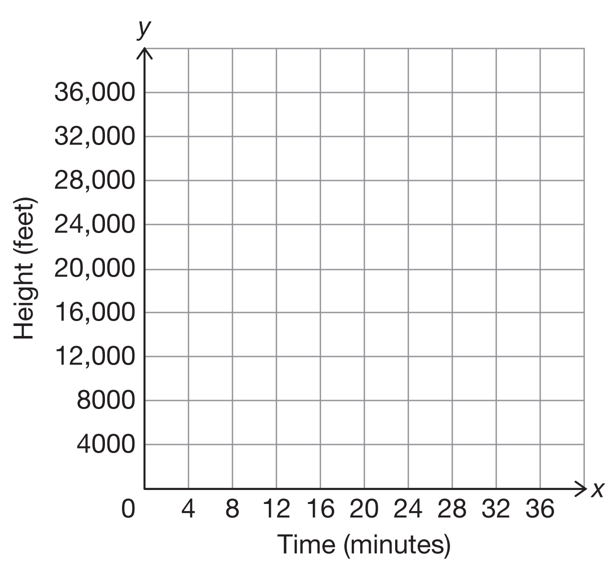
3. Write a function, $g(t)$, to represent this problem situation.

4. Complete the table shown. First, determine the unit of measure for each expression. Then, describe the contextual meaning of each part of the function. Finally, choose a term from the word box to describe the mathematical meaning of each part of the function.

input value	output value	rate of change
y-intercept	x-intercept	

		Description	
Expression	**Units**	**Contextual Meaning**	**Mathematical Meaning**
t			
-1500			
$-1500t$			
36,000			
$-1500t + 36,000$			

5. Graph $g(t)$ on the coordinate plane shown.



You have just represented the *As We Make Our Final Descent* scenario in different ways:

- numerically, by completing a table,
- algebraically, by writing a function, and
- graphically, by plotting points.

Let's consider how to use each of these representations to answer questions about the problem situation.

2

6. Determine how long will it take the plane to descend to 14,000 feet.

a. Use the table to determine how long it will take the plane to descend to 14,000 feet.

b. Graph and label $y = 14{,}000$ on the coordinate plane. Then determine the intersection point. Explain what the intersection point means in terms of this problem situation.

c. Substitute 14,000 for $g(t)$ and solve the equation for t. Interpret your solution in terms of this problem situation.

d. Compare and contrast your solutions using the table, graph, and the function. What do you notice? Explain your reasoning.

7. Determine how long it will take the plane to descend to 24,000 feet.

   a. Use the table to determine how long it will take the plane to descend to 24,000 feet.

   b. Graph and label $y = 24{,}000$ on the coordinate plane. Then determine the intersection point. Explain what the intersection point means in terms of this situation.

   c. Substitute 24,000 for $g(t)$ and solve the equation for t. Interpret your solution in terms of this situation.

   d. Compare and contrast your solutions using the table, graph, and the function. What do you notice? Explain your reasoning.

8. For how many heights can you calculate the *exact* time using the:

a. table?

2

b. graph?

c. function?

9. Use the word bank to complete each sentence.

always	sometimes	never

If I am given a dependent value and need to calculate an independent value of a linear function,

a. I can ______________________ use a table to determine an *approximate* value.

b. I can ______________________ use a table to calculate an *exact* value.

c. I can ______________________ use a graph to determine an *approximate* value.

d. I can ______________________ use a graph to calculate an *exact* value.

e. I can ______________________ use a function to determine an *approximate* value.

f. I can ______________________ use a function to calculate an *exact* value.

PROBLEM 2 Making the Exchange

The plane has landed in the United Kingdom and the Foreign Language Club is ready for their adventure. Each student on the trip boarded the plane with £300. They each brought additional U.S. dollars with them to exchange as needed. The exchange rate from U.S. dollars to British pounds is £0.622101 pound to every dollar.

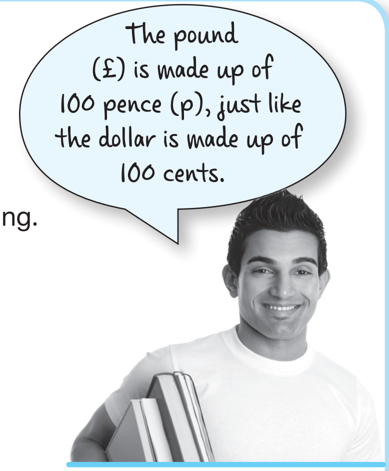
1. Write a function to represent the total amount of money in British pounds each student will have after exchanging additional U.S. currency. Define your variables.

2. Identify the slope and interpret its meaning in terms of this problem situation.

3. Identify the y-intercept and interpret its meaning in terms of this problem situation.

4. Dawson would like to exchange \$70 more.

Jonathon thinks Dawson should have a total of £343.54707. Erin says he should have a total of £343.55, and Tre says he should have a total of £342. Who's correct? Who's reasoning is correct? Why are the other students not correct? Explain your reasoning.



Jonathon

$f(d) = 300 + 0.622101d$

$f(d) = 300 + 0.622101(70)$

$f(d) = 300 + 43.54707$

$f(d) = 343.54707$

Erin

$f(d) = 300 + 0.622101d$

$f(d) = 300 + 0.622101(70)$

$f(d) = 300 + 43.54707$

$f(d) = 343.54707$

$f(d) \approx 343.55$

Tre

$f(d) = 300 + 0.6d$

$f(d) = 300 + 0.6(70)$

$f(d) = 300 + 42$

$f(d) = 342$

2

5. How many total pounds will Dawson have if he only exchanges an additional \$50? Show your work.

PROBLEM 3 Using Technology to Complete Tables

Throughout this lesson you used multiple representations and paper-and-pencil to answer questions. You can also use a graphing calculator to answer questions. Let's first explore how to use a graphing calculator to create a table of values for converting U.S. dollars to British pounds.

You can use a graphing calculator to complete a table of values for a given function.

The exchange function is $f(d) = 300 + 0.622101d$.

Step 1: Press **Y=**

Step 2: Enter the function. Press **ENTER**.

Step 3: Press **2ND TBLSET** (above **WINDOW**).

TblStart is the starting data value for your table. Enter this value.

ΔTbl (read "delta table") is the increment. This value tells the table what intervals to count by for the independent quantity. If **ΔTbl** = 1 then the values in your table would go up by 1s. If **ΔTbl** = -1, the values would go *down* by 1s. Enter the **ΔTbl**.

For this scenario, you will not exchange any currency less than $100. Set the **TblStart** value to 100.

Step 4: Press **2ND TABLE** (above **GRAPH**). Use the up and down arrows to scroll through the data.

1. Use your graphing calculator and the **TABLE** feature to complete the table shown.

U.S. Currency	British Currency
$	£
100	
150	
175	
	455.53
	466.10

Analyze the given U.S. Currency dollar amounts and decide how to set the increments for ΔTbl.

2. Were you able to complete the table using the **TABLE** feature? Why or why not? What adjustments, if any, can you make to complete the table?

2

PROBLEM 4 Using Technology to Analyze Graphs

There are several graphing calculator strategies you can use to analyze graphs to answer questions. Let's first explore the **value** feature. This feature works well when you are given an independent value and want to determine the corresponding dependent value.

You can use the **value** feature on a graphing calculator to determine an exact data value on a graph.

Step 1: Press **Y=**. Enter your function.

Step 2: Press **WINDOW**. Set appropriate values for your function. Then press **GRAPH**.

Step 3: Press **2ND** and then **CALC**. Select **1:value**. Press **ENTER**. Then type the given independent value next to **X=** and press **ENTER**. The cursor moves to the given independent value and the corresponding dependent value is displayed at the bottom of the screen.

Be sure to double check that you typed in the correct function.

If you get an error message, go back and adjust your WINDOW.
ERR:INVALID
1: Quit
2: Goto

Use the **value** feature to answer each question.

1. How many total British pounds will Amy have if she exchanges an additional:

   a. \$375?

   b. \$650?

   c. \$2000

2. How can you verify that each solution is correct?

3. What are the advantages and limitations of using the **value** feature?

2

Let's now explore the **intersect** feature of **CALC**. You can use this feature to determine an independent value when given a dependent value.

Suppose you know that Jorge has a total of £725.35. You can first write this as $f(d) = 300 + 0.622101d$ and $y = 725.35$. Then graph each equation, calculate the intersection point, and determine the additional amount of U.S. currency that Jorge exchanged.

You can use the **intersect** feature to determine an independent value when given a dependent value.

Step 1: Press **Y=**. Enter the two equations, one next to Y_1= and one next to Y_2=.

Step 2: Press **WINDOW**. Set appropriate bounds so you can see the intersection of the two equations. Then press **GRAPH**.

Step 3: Press **2ND CALC** and then select **5:intersect**. The cursor should appear somewhere on one of the graphs, and at the bottom of the screen you will see **First curve?** Press **ENTER**.
The cursor should then move to somewhere on the other graph, and you will see **Second curve?** Press **ENTER**.
You will see **Guess?** at the bottom of the screen. Move the cursor to where you think the intersection point is and Press **ENTER**.
The intersection point will appear.

You can use your arrow keys to scroll to different features.

Use the **intersect** feature to answer each question.

4. How many additional U.S. dollars did Jorge exchange if he has a total of:

   a. £725.35?

   b. £1699.73?

2

5. How can you verify that each solution is correct?

6. What are the advantages and limitations of the intersect feature?

7. Do you think you could use each of the graphing calculator strategies discussed in this lesson with any function, not just linear functions?

PROBLEM 5 Graphing Calculator Practice

Use a graphing calculator to evaluate each function. Explain the strategy you used.

1. $f(x) = 14.95x + 31.6$

   a. $f(3.5)$

   b. $f(16.37)$

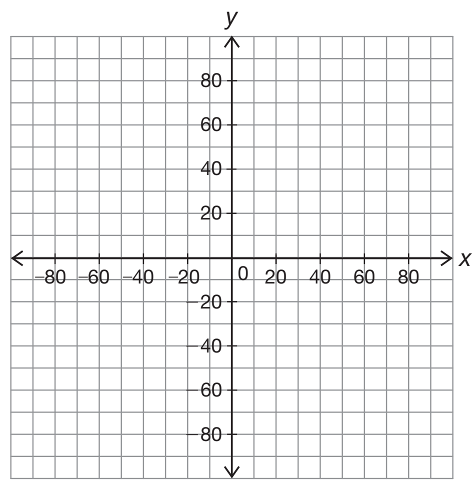
   c. $f(50.1)$

2.

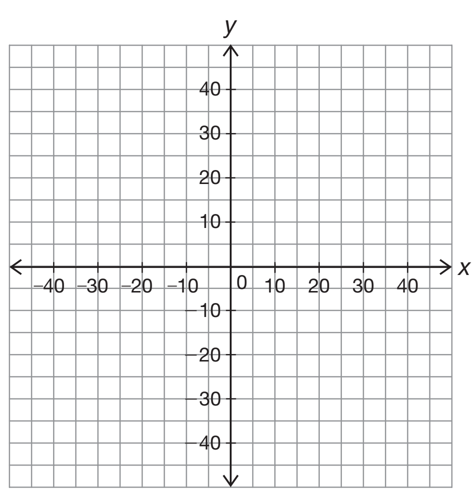
x	$-\frac{7}{9}x - 18$
$-\frac{1}{2}$	
0	
$-4\frac{1}{2}$	

3. Use a graphing calculator and the **intersect** feature to determine each independent value. Then sketch the graphs on the coordinate plane provided.

a. $f(x) = -3.315x - 20$ when $f(x) = 23.38$



b. $\frac{1}{2}x + 5 = 16\frac{4}{5}$



4. Use the word box to complete each sentence, and then explain your reasoning.

always	sometimes	never

If I am using a graphing calculator and I am given a dependent value and need to calculate an independent value,

a. I can ______________ use a table to determine an *approximate* value.

b. I can ______________ use a table to calculate an *exact* value.

c. I can ______________ use a graph to determine an *approximate* value.

d. I can ______________ use a graph to calculate an exact value.

e. I can ______________ use a function to determine an approximate value.

f. I can ______________ use a function to calculate an exact value.

Be prepared to share your solutions and methods.

2.3

Scouting for Prizes!

Modeling Linear Inequalities

LEARNING GOALS

In this lesson, you will:

- Write and solve inequalities.
- Analyze a graph on a coordinate plane to solve problems involving inequalities.
- Interpret how a negative rate affects how to solve an inequality.

KEY TERM

- solve an inequality

Scouting began in 1907 by a Lieutenant General in the British Army, Robert Baden-Powell, as a way to teach young men and women about different outdoor activities and survival techniques. While he was a military officer, Baden-Powell taught his soldiers how to survive in the wilderness and spent much time on scouting missions in enemy territory. He became a national hero during this time which helped fuel the sales of a book he had written, *Aids to Scouting*. When he returned home many people wanted him to rewrite his book for boys. While his rewritten book, *Scouting for Boys*, contained many of the same ideas about outdoor living, he left out the military aspects of his first book. Boys immediately began forming their own Scout patrols and wrote to Baden-Powell asking for his assistance. The Scouting movement has been growing and changing ever since.

Do you think wilderness survival skills are necessary today? If yes, why do you think we still need these skills? If no, why do you think people still learn them if they are unnecessary?

2

PROBLEM 1 Popcorn Pays Off

Alan's camping troop is selling popcorn to earn money for an upcoming camping trip. Each camper starts with a credit of $25 toward his sales, and each box of popcorn sells for $3.75.

Alan can also earn bonus prizes depending on how much popcorn he sells. The table shows the different prizes for each of the different sales levels. Each troop member can choose any one of the prizes at or below the sales level.

Sales (dollars)	Gift Cards (2 of each value)	Bonus Prizes
$250	$10	
$350	$15	
$450	$20	
$600		Cyclone Sprayer
$650	$30	
$850	$40	
$1100	$55	
$1300	$75	
$1500		Choose your prize!
$1800	$110	
$2300	$150	
$2500		6% toward college scholarship

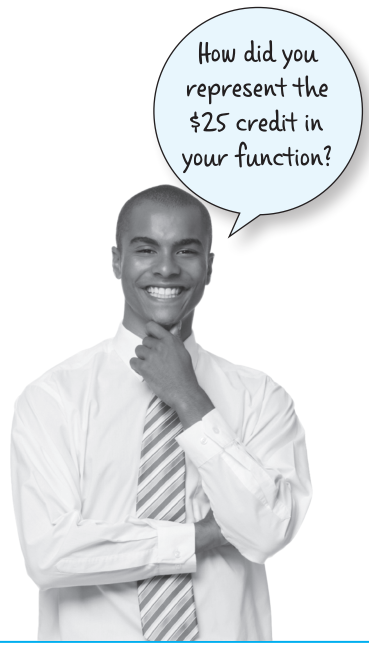
1. Write a function, $f(b)$, to show Alan's total sales as a function of the number of boxes of popcorn he sells.

2. Analyze the function you wrote.

   a. Identify the independent and dependent quantities and their units.

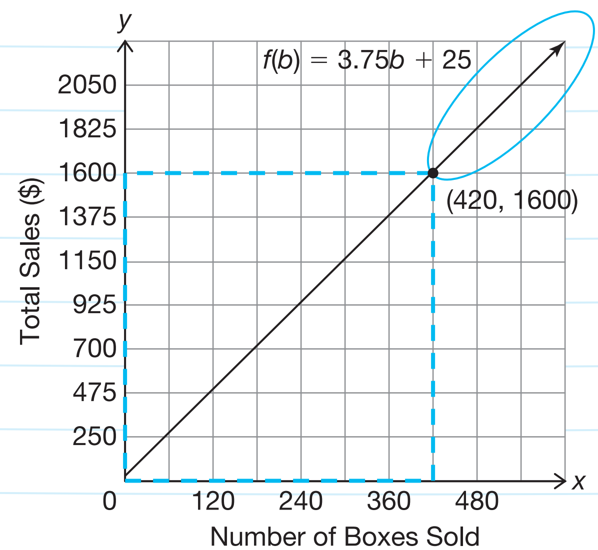
   b. What is the rate of change and what does it represent in this problem situation?

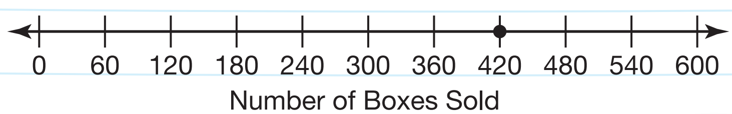
   c. What is the y-intercept and what does it represent in this problem situation?



Now, let's analyze your function represented on a graph.

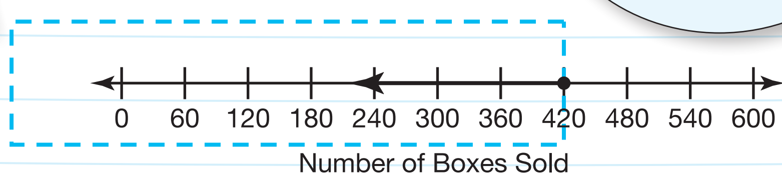
The graph shown represents the change in the total sales as a function of boxes sold. The oval and box represent the total sales at specific intervals.



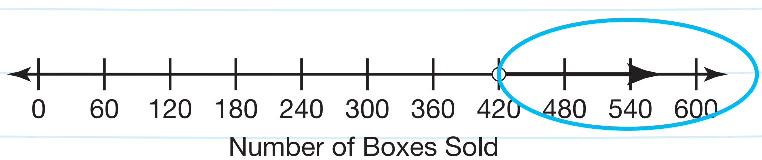


The point at (420, 1600) means that at 420 boxes sold, the total sales is equal to $1600. This is represented on the number line as a closed point at 420. When $f(b) = 1600$, then $b = 420$.

The box represents all the numbers of boxes sold, b, that would earn Alan $1600 or less. When $f(b) \leq 1600$ then $b \leq 420$.



The oval represents all the numbers of boxes sold, b, that would earn Alan more than $1600. When $f(b) > 1600$, then $b > 420$.



3. Explain the difference between the open and closed circles on the number lines.

4. Use the graph to answer each question. Write an equation or inequality statement for each.

   a. How many boxes would Alan have to sell to earn at least $925?

   b. How many boxes would Alan have to sell to earn less than $2050?

   c. How many boxes would Alan have to sell to earn exactly $700?

PROBLEM 2 What's Your Strategy—Your Algebraic Strategy?

Another way to determine the solution set of an inequality is to solve it algebraically. To **solve an inequality** means to determine the values of the variable that make the inequality true. The objective when solving an inequality is similar to the objective when solving an equation: You want to isolate the variable on one side of the inequality symbol.

2

In order to earn two $55 gift cards, Alan's total sales, $f(b)$, needs to be at least $1100. You can set up an inequality and solve it to determine the number of boxes Alan needs to sell.

$$f(b) \geq 1100$$

$$3.75b + 25 \geq 1100$$

Solve the inequality in the same way you would solve an equation.

$$3.75b + 25 \geq 1100$$

$$3.75b + 25 - 25 \geq 1100 - 25$$

$$3.75b \geq 1075$$

$$\frac{3.75b}{3.75} \geq \frac{1075}{3.75}$$

$$b \geq 286.66\ldots$$

Alan needs to sell at least 287 boxes of popcorn to earn two $55 gift cards.

1. Why was the answer rounded to 287?

2. Write and solve an inequality for each. Show your work.

   a. What is the greatest number of boxes Alan could sell and still not have enough to earn the Cyclone Sprayer?

b. At least how many boxes would Alan have to sell to be able to choose his own prize?

2

PROBLEM 3 Reversing the Sign

Alan's camping troop hikes down from their campsite at an elevation of 4800 feet to the bottom of the mountain. They hike down at a rate of 20 feet per minute.

1. Write a function, $h(m)$, to show the troop's elevation as a function of time in minutes.

2. Analyze the function.

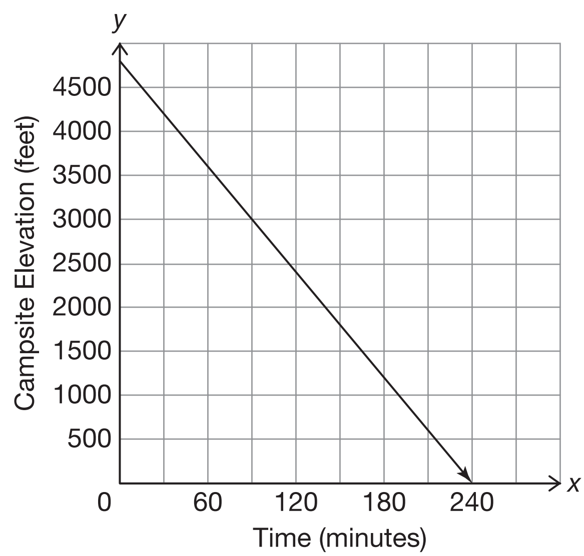
   a. Identify the independent and dependent quantities and their units.

   b. Identify the rate of change and explain what it means in terms of this problem situation.

   c. Identify the y-intercept and explain what it means in terms of this problem situation.

   d. What is the x-intercept and explain what it means in terms of this problem situation?

3. Label the function on the coordinate plane.



4. Use the graph to determine how many minutes passed if the troop is below 3200 feet. Draw an oval on the graph to represent this part of the function and write the corresponding inequality statement.

5. Write and solve an inequality to verify the solution set you interpreted from the graph.

6. Compare and contrast your solution sets using the graph and the function. What do you notice?

7. Complete the table by writing the corresponding inequality statement that represents the number of minutes for each height.

$h(m)$	m
$h(m) > 3200$	
$h(m) \geq 3200$	
$h(m) = 3200$	
$h(m) < 3200$	
$h(m) \leq 3200$	

a. Compare each row in the table shown. What do you notice about the inequality signs?

b. Explain your answer from part (a). Use what you know about solving inequalities when you have to multiply or divide by a negative number.

Talk the Talk

2

1. Explain the differences when you solved inequalities involving the function $f(b) = 3.75b + 25$ and the function $h(m) = -20m + 4800$.

2. Solve each inequality and then graph the solution on the number line.

   a. $-\frac{2}{3}x \geq 7$

   b. $32 > 23 - x$

   c. $2(x + 6) < 10$

Be prepared to share your solutions and methods.

2.4

We're Shipping Out!

Solving and Graphing Compound Inequalities

LEARNING GOALS

In this lesson, you will:

- Write simple and compound inequalities.
- Graph compound inequalities.
- Solve compound inequalities.

KEY TERMS

- compound inequality
- solution of a compound inequality
- conjunction
- disjunction

How many different ways do you think water exists? You may instantly think of water in a liquid state like you see in raindrops, or in lakes, ponds, or oceans. However, you probably also know that water can be a solid like hail, or ice cubes; or as a gas as in the humidity you may feel on a hot summer day, or the steam you see. What factors do you think play a role in the way water exists? Can you think of other things that can take the form of a solid, liquid, *and* gas?

2

PROBLEM 1 GoodSportsBuys.com

GoodSportsBuys.com is an online store that offers discounts on sports equipment to high school athletes. When customers buy items from the site, they must pay the cost of the items as well as a shipping fee. At GoodSportsBuys.com, a shipping fee is added to each order based on the total cost of all the items purchased. This table provides the shipping fee categories for GoodSportsBuys.com.

Total Cost of Items	Shipping Fee
$0.01 up to and including $20	$6.50
More than $20 up to and including $50	$9.00
Between $50 and $75	$11.00
From $75 up to, but not including, $100	$12.25
$100 or more	$13.10

1. What is the least amount a customer can spend on items and pay $6.50 for shipping?

2. What is the greatest amount a customer can spend on items and pay $6.50 for shipping?

3. What is the shipping fee if Sarah spends exactly $75.00 on items? Explain your reasoning.

4. Harvey says he will spend $13.10 on shipping fees if he spends exactly $100 on items. Is he correct? Explain your reasoning.

5. Consider the table of shipping costs to complete each statement using the phrase "greater than," "less than," "greater than or equal to," or "less than or equal to."

   a. You will pay $6.50 in shipping fees if you spend:

   b. You will pay $9.00 in shipping fees if you spend:

   c. You will pay $11.00 in shipping fees if you spend:

   d. You will pay $12.25 in shipping fees if you spend:

   e. You will pay $13.10 in shipping fees if you spend:

A **compound inequality** is an inequality that is formed by the union, "or," or the intersection, "and," of two simple inequalities.

6. You can use inequalities to represent the various shipping fee categories at GoodSportsBuys.com. If you let x represent the total cost of items purchased, you can write an inequality to represent each shipping fee category. Complete each inequality using an inequality symbol.

   a. $6.50 shipping fees: x ☐ $0.01 and x ☐ $20

   b. $9.00 shipping fees: x ☐ $20 and x ☐ $50

   c. $11.00 shipping fees: x ☐ $50 and x ☐ $75

   d. $12.25 shipping fees: x ☐ $75 and x ☐ $100

   e. $13.10 shipping fees: x ☐ $100

7. Identify the inequalities in Question 6 that are compound inequalities.

Let's consider two examples of compound inequalities.

$x > 2$ and $x \le 7$

This inequality is read as "all numbers greater than 2 and less than or equal to 7." This inequality can also be written in the compact form of $2 < x \le 7$.

$x \le -4$ or $x > 2$

This inequality is read as "all numbers less than or equal to -4 *or* greater than 2."

Only compound inequalities containing "and" can be written in compact form.

8. Write the compound inequalities from Question 6 using the compact form.

a. $6.50 shipping fees: ________________

b. $9.00 shipping fees: ________________

c. $11.00 shipping fees: ________________

d. $12.25 shipping fees: ________________

PROBLEM 2 More Than One Solution

1. Water becomes non-liquid when it is 32°F or below, or when it is at least 212°F.

   a. Represent this information on a number line.

   b. Write a compound inequality to represent the same information. Define your variable.

2. Luke and Logan play for the same baseball team. They practice at the Lions Park baseball field. Luke lives 3 miles from the field, and Logan lives 2 miles from the field.

   a. First, plot a point to represent the location of the Lions Park baseball field.

   b. Next, use your point that represents Lions Park, and draw a circle to represent all the possible places Luke could live.

   c. Finally, use your point that represents Lions Park, and draw another circle to represent all the possible places Logan could live.

   d. What is the shortest distance, d, that could separate their homes?

   e. What is the longest distance, d, that could separate their homes?

f. Write a compound inequality to represent all the possible distances that could separate their homes.

g. Represent the solution on a number line.

2

3. Jodi bought a new car with a 14-gallon gas tank. Around town she is able to drive 336 miles on one tank of gas. On her first trip traveling on highways, she drove 448 miles on one tank of gas. What is her average miles per gallon around town? What is her average miles per gallon on highways?

a. Write a compound inequality that represents how many miles Jodi can drive on a tank of gas. Let *m* represents the number of miles per gallon of gas.

b. Rewrite the compound inequality as two simple inequalities separated by either "and" or "or."

c. Solve each simple inequality.

d. Go back to the compound inequality you wrote in Question 3, part (a). How can you solve the compound inequality without rewriting it as two simple inequalities? Solve the compound inequality.

e. Compare the solution you calculated in Question 3, part (c) with the solution you calculated in Question 3, part (d). What do you notice?

f. Explain your solution in terms of the problem situation.

g. Represent the solution on a number line. Describe the shaded region in terms of the problem situation.

PROBLEM 3 Solving Compound Inequalities

Remember, a compound inequality is an inequality that is formed by the union, "or," or the intersection, "and," of two simple inequalities.

The **solution of a compound inequality** in the form $a < x < b$, where a and b are any real numbers, is the part or parts of the solutions that satisfy both of the inequalities. This type of compound inequality is called a **conjunction**. The solution of a compound inequality in the form $x < a$ or $x > b$, where a and b are any real numbers, is the part or parts of the solution that satisfy either inequality. This type of compound inequality is called a **disjunction**.

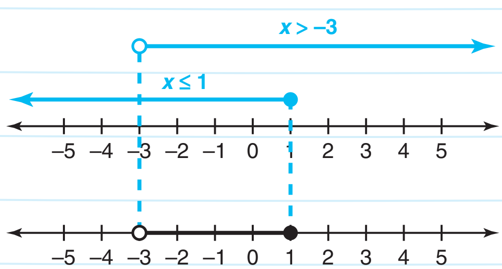
1. Classify each solution to all the questions in Problem 2 as either a conjunction or disjunction.

Let's consider two examples for representing the solution of a compound inequality on a number line.

The compound inequality shown involves "and" and is a conjunction.

$x \leq 1$ and $x > -3$

Represent each part above the number line.



$x \leq 1$ and $x > -3$

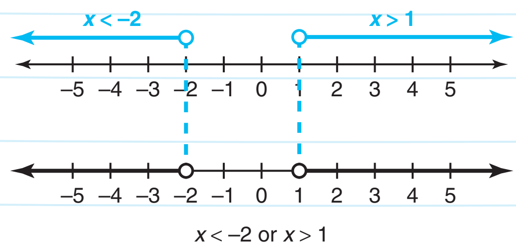
$-3 < x \leq 1$

The solution is the region that satisfies both inequalities. Graphically, the solution is the overlapping, or the intersection, of the separate inequalities.

The compound inequality shown involves "or" and is a disjunction.

$$x < -2 \text{ or } x > 1$$

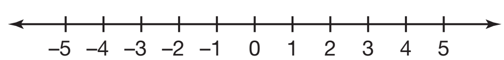
Represent each part above the number line.



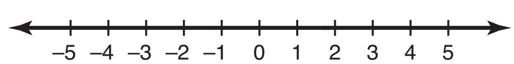
The solution is the region that satisfies either inequality. Graphically, the solution is the union, or all the regions, of the separate inequalities.

2. Consider the two worked examples in a different way.

a. If the compound inequality in the first worked example was changed to the disjunction, $x \leq 1$ or $x > -3$, how would the solution set change? Explain your reasoning.

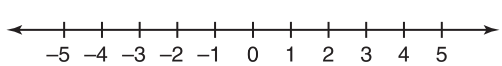


b. If the compound inequality in the second worked example was changed to the conjunction, $x < -2$ or $x > 1$, how would the solution set change? Explain your reasoning.

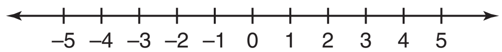


3. Represent the solution to each compound inequality on the number line shown. Then, write the final solution that represents the graph.

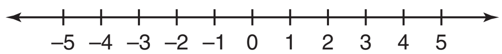
a. $x < 2$ or $x > 3$



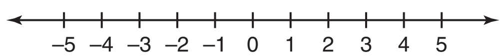
b. $-1 \geq x \geq -1$



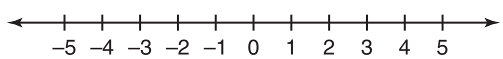
c. $x < 0$ or $x < 2$



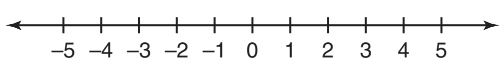
d. $x > 1$ and $x < -2$



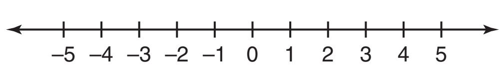
e. $x < 3$ and $x > 2$



f. $x < 2$ and $x < -1$



g. $x > -1$ or $x < 0$



To solve a compound inequality written in compact form, isolate the variable between the two inequality signs, and then graph the resulting statement. To solve an inequality involving "or," simply solve each inequality separately, keeping the word "or" between them, and then graph the resulting statements.

4. Solve and graph each compound inequality showing the steps you performed. Then, write the final solution that represents the graph.

   a. $6 < x - 6 \leq 9$

   b. $-2 < -x < 6$

   c. $-4 \leq -3x + 1 \leq 12$

   d. $2x + 7 < 10$ or $-2x + 7 > 10$

e. $\frac{1}{2}x + 3 > 4$ or $-x < 3$

f. $1 + 6x > 11$ or $x - 4 < -5$

Be prepared to share your solutions and methods.

2

2.5

Play Ball!

Absolute Value Equations and Inequalities

LEARNING GOALS

In this lesson, you will:

- Understand and solve absolute values.
- Solve linear absolute value equations.
- Solve and graph linear absolute value inequalities on number lines.
- Graph linear absolute values and use the graph to determine solutions.

KEY TERMS

- opposites
- absolute value
- linear absolute value equation
- linear absolute value inequality
- equivalent compound inequalities

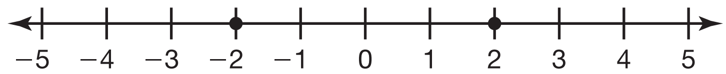
All games and sports have specific rules and regulations. There are rules about how many points each score is worth, what is in-bounds and what is out-of-bounds, and what is considered a penalty. These rules are usually obvious to anyone who watches a game. However, some of the regulations are not so obvious. For example, the National Hockey League created a rule that states that a blade of a hockey stick cannot be more than three inches or less than two inches in width at any point. In the National Football League, teams that wear black shoes must wear black shoelaces and teams that wear white shoes must wear white laces. In the National Basketball Association, the rim of the basket must be a circle exactly 18 inches in diameter. Most sports even have rules about how large the numbers on a player's jersey can be!

Do you think all these rules and regulations are important? Does it really matter what color a player's shoelaces are? Why do you think professional sports have these rules, and how might the sport be different if these rules did not exist?

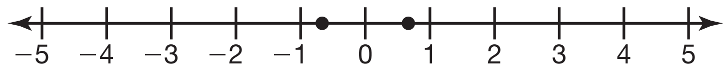
PROBLEM 1 Opposites Attract? Absolutely!

1. Analyze each pair of numbers and the corresponding graph.

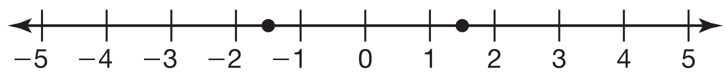
 a. -2 and 2



 b. $-\frac{2}{3}$ and $\frac{2}{3}$



 c. -1.5 and 1.5



2. Describe the relationship between the two numbers.

3. What do you notice about the distance each point lies away from zero on each number line?

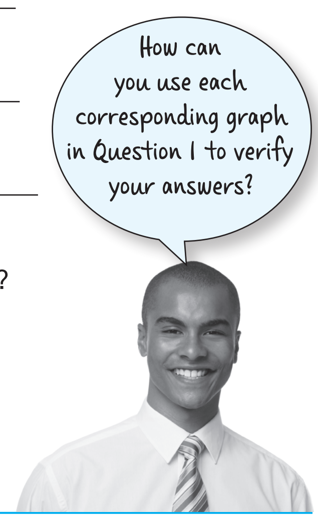
Two numbers that are an equal distance, but are in different directions, from zero on the number line are called **opposites**. The **absolute value** of a number is its distance from zero on the number line.

4. Write each absolute value.

 a. $|-2| =$ __________ $|2| =$ __________

 b. $\left|-\frac{2}{3}\right| =$ __________ $\left|\frac{2}{3}\right| =$ __________

 c. $|-1.5| =$ __________ $|1.5| =$ __________



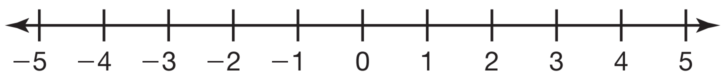
5. What do you notice about each set of answers for Question 4?

6. Determine the value of each. Show your work.

a. $|3 - 8|$ **b.** $|3| - |8|$ **c.** $|4(5)|$

d. $|-4| \cdot |5|$ **e.** $\left|\frac{12}{-3}\right|$ **f.** $\frac{|12|}{|-3|}$

7. Determine the solution(s) to each equation.



a. $x = 5$ **b.** $|x| = 5$

c. $|x| = -5$ **d.** $|x| = 0$

8. Analyze each equation containing an absolute value symbol in Question 7. What does the form of the equation tell you about the possible number of solutions?

PROBLEM 2 Too Heavy? Too Light? You're Out!

The official rules of baseball state that all baseballs used during professional games must be within a specified range of weights. The baseball manufacturer sets the target weight of the balls at 145.045 grams on its machines. The specified weight allows for a difference of 3.295 grams. This means the weight can be 3.295 grams greater than or less than the target weight.

2

1. Write an expression to represent the difference between a manufactured baseball's weight and the target weight. Use *w* to represent a manufactured baseball's weight.

2. Suppose the manufactured baseball has a weight that is greater than the target weight.
   a. Write an equation to represent the greatest acceptable difference in the weight of a baseball.

   b. Solve your equation to determine the greatest acceptable weight of a baseball.

3. Suppose the manufactured baseball has a weight that is less than the target weight.
   a. Write an equation to represent the least acceptable difference in weight.

   b. Solve your equation to determine the least acceptable weight of a baseball.

The two equations you wrote can be represented by the **linear absolute value equation** $|w - 145.045| = 3.295$. In order to solve any absolute value equation, recall the definition of absolute value.

Consider this linear absolute value equation.

$$|a| = 6$$

There are two points that are 6 units away from zero on the number line: one to the right of zero, and one to the left of zero.

$$+(a) = 6 \quad \text{or} \quad -(a) = 6$$

$$a = 6 \quad \text{or} \quad a = -6$$

Now consider the case where $a = x - 1$.

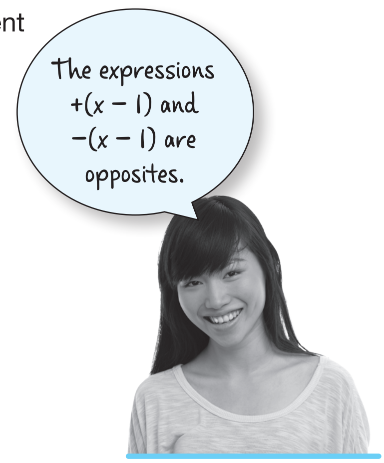
$$|x - 1| = 6$$

If you know that $|a| = 6$ can be written as two separate equations, you can rewrite any absolute value equation.

$$+(a) = 6 \quad \text{or} \quad -(a) = 6$$

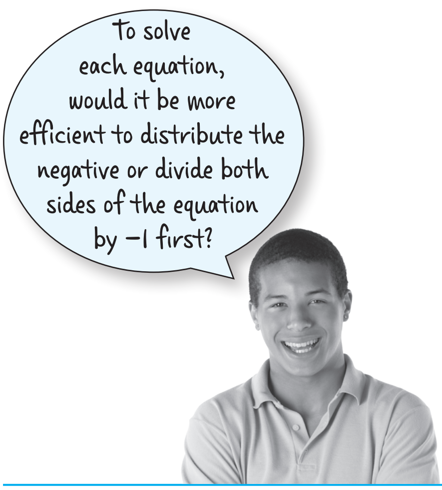
$$+(x - 1) = 6 \quad \text{or} \quad -(x - 1) = 6$$

4. How do you know the expressions $+(a)$ and $-(a)$ represent opposite distances?

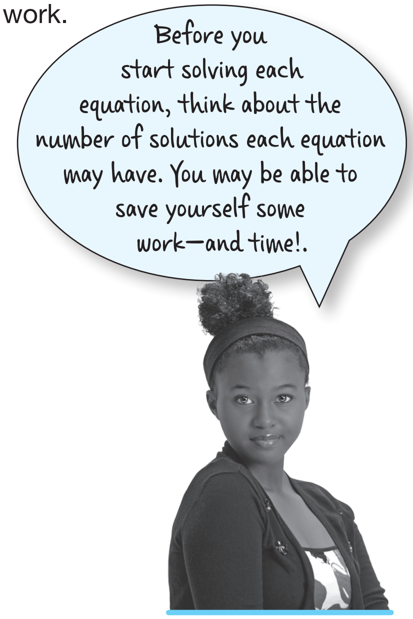


5. Determine the solution(s) to the linear absolute value equation $|x - 1| = 6$. Then check your answer.

$+(x - 1) = 6$ $\qquad$ $-(x - 1) = 6$



6. Solve each linear absolute value equation. Show your work.



a. $|x + 7| = 3$

2

b. $|x - 9| = 12$

c. $|3x + 7| = -8$

d. $|2x + 3| = 0$

7. Cho, Steve, Artie, and Donald each solved the equation $|x| - 4 = 5$.

Artie

$$|x| - 4 = 5$$

$(x) - 4 = 5$ | $-(x) - 4 = 5$

$(x) = 9$ | $-x = 9$

$x = -9$

Donald

$$|x| - 4 = 5$$

$$|x| = 9$$

$(x) = 9$ | $-(x) = 9$

$x = -9$

Cho

$$|x| - 4 = 5$$

$(x) - 4 = 5$ | $-[(x) - 4] = 5$

$x - 4 = 5$ | $-x + 4 = 5$

$x = 9$ | $-x = 1$

$x = -1$

Steve

$$|x| - 4 = 5$$

$(x) - 4 = +5$ | $-(x) - 4 = -5$

$x = 9$ | $-x - 4 = -5$

$-x = -1$

$x = 1$

a. Explain how Cho and Steve incorrectly rewrote the absolute value equation as two separate equations.

b. Explain the difference in the strategies that Artie and Donald used. Which strategy do you prefer? Why?

8. Solve each linear absolute value equation.

a. $|x| + 16 = 32$

b. $23 = |x - 8| + 6$

c. $3|x - 2| = 12$

d. $35 = 5|x + 6| - 10$

PROBLEM 3 Too Big? Too Small? Just Right.

In *Too Heavy? Too Light? You're Out!* you determined the linear absolute value equation to identify the most and least a baseball could weigh and still be within the specifications. The manufacturer wants to determine all of the acceptable weights that the baseball could be and still fit within the specifications. You can write a **linear absolute value inequality** to represent this problem situation.

2

1. Write a linear absolute value inequality to represent all baseball weights that are within the specifications.

2. Determine if each baseball has an acceptable weight. Explain your reasoning.

   a. A manufactured baseball weighs 147 grams.

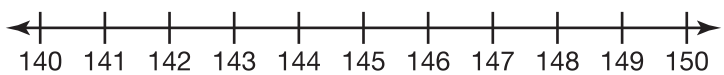
   b. A manufactured baseball weighs 140.8 grams.

   c. A manufactured baseball weighs 148.34 grams.

d. A manufactured baseball weighs 141.75 grams.

3. Complete the inequality to describe all the acceptable weights, where w is the baseball's weight. Then use the number line to graph this inequality.

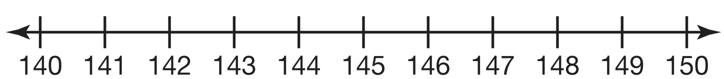
$\_\_\_\_\_\_\_\_ \leq w \leq \_\_\_\_\_\_\_\_$



4. Raymond has the job of disposing of all baseballs that are not within the acceptable weight limits.

a. Write an inequality to represent the weights of baseballs that Raymond can dispose of.

b. Graph the inequality on the number line.



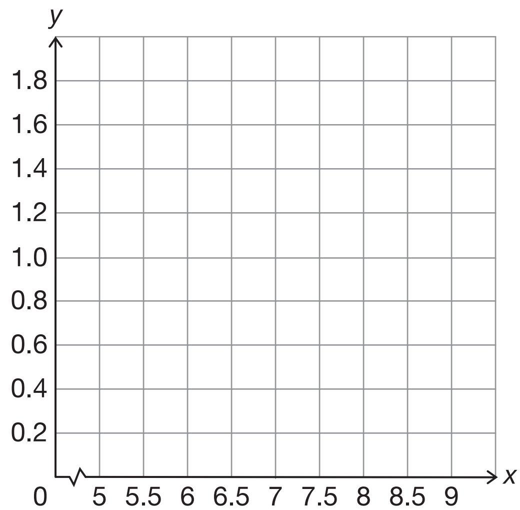
In Little League Baseball, the diameter of the ball is slightly smaller than that of a professional baseball.

5. The same manufacturer also makes Little League baseballs. For these baseballs, the manufacturer sets the target diameter to be 7.47 centimeters. The specified diameter allows for a difference of 1.27 centimeters.

a. Denise measures the diameter of the Little League baseballs as they are being made. Complete the table to determine each difference. Then write the linear absolute value expression used to determine the diameter differences.

	Independent Quantity	Dependent Quantity
	Diameter of the Little League Baseballs	**Target and Actual Diameter Difference**
Units		
	6.54	
	8.75	
	7.39	
	5.99	
	8	
	9.34	
	7.47	
	d	

b. Graph the linear absolute value function, $f(d)$, on a graphing calculator. Sketch the graph on the coordinate plane.



6. Determine the diameters of all Little League baseballs that fit within the specifications.

a. Use your graph to estimate the diameters of all the Little League baseballs that fit within the specifications. Explain how you determined your answer.

b. Algebraically determine the diameters of all the baseballs that fit within the specification. Write your answer as an inequality.

7. The manufacturer knows that the closer the diameter of the baseball is to the target, the more likely it is to be sold. The manufacturer decides to only keep the baseballs that are less than 0.75 centimeter from the target diameter.

   a. Algebraically determine which baseballs will not fall within the new specified limits and will not be kept. Write your answer as an inequality.

   b. How can you use your graph to determine if you are correct?

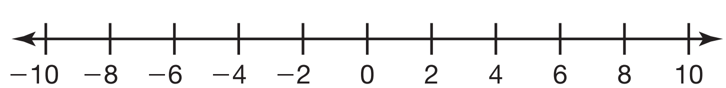
Talk the Talk

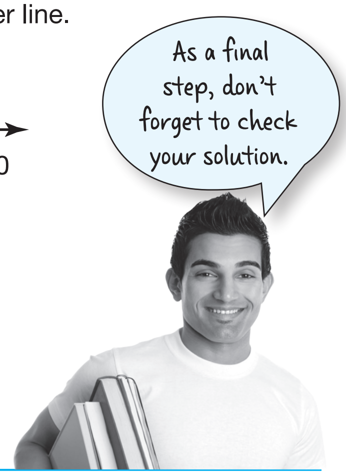
Absolute value inequalities can take four different forms as shown in the table. To solve a linear absolute value inequality, you must first write it as an **equivalent compound inequality**.

Absolute Value Inequality	Equivalent Compound Inequality
$\lvert ax + b\rvert < c$	$-c < ax + b < c$
$\lvert ax + b\rvert \le c$	$-c \le ax + b \le c$
$\lvert ax + b\rvert > c$	$ax + b < -c$ or $ax + b > c$
$\lvert ax + b\rvert \ge c$	$ax + b \le -c$ or $ax + b \ge c$

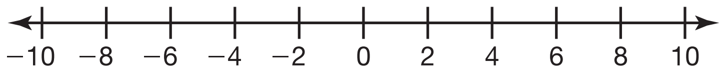
1. Solve the linear absolute value inequality by rewriting it as an equivalent compound inequality. Then graph your solution on the number line.

   a. $\lvert x + 3\rvert < 4$





b. $6 \leq |2x - 4|$



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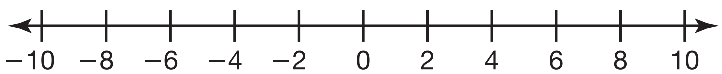
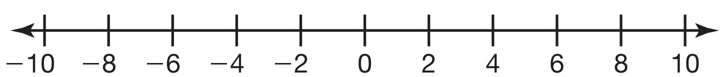
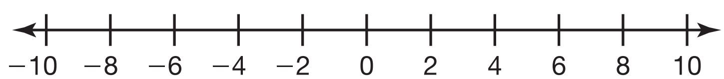
c. $|-5x + 8| + 2 < 25$



d. $|x + 5| > -1$



e. $|x + 5| < -1$



Be prepared to share your solutions and methods.

2

2.6

Choose Wisely!

Understanding Non-Linear Graphs and Inequalities

LEARNING GOALS

In this lesson, you will:

- Identify the appropriate function to represent a problem situation.
- Determine solutions to linear functions using intersection points.
- Determine solutions to non-linear functions using intersection points.
- Describe advantages and disadvantages of using technology different methods to solve functions with and without technology.

We make decisions constantly: what time to wake up, what clothes to wear to school, whether or not to eat a big or small breakfast. And those decisions all happen a few hours after you wake up! So how do we decide what we do? There are actually a few different techniques for making decisions. One technique, which you have most likely heard about from a teacher, is weighing the pros and cons of your options then choosing the one that will result in the best outcome. Another technique is called satisficing—which means just using the first acceptable option, which probably isn't the best technique. Have you ever flipped a coin to make a decision? That is called flipism. Finally, some people may follow a person they deem an "expert" while others do the most opposite action recommended by "experts." While the technique you use isn't really important for some decisions (flipping a coin to decide whether or not to watch a TV show), there are plenty of decisions where there is a definite better choice (do you really want to flip a coin to decide whether to wear your pajamas to school?). The best advice for making decisions is to know your goal, gather all the information you can, determine pros and cons of each alternative decision, and make the decision.

What technique do you use when making decisions? Do you think some people are better decision makers than others? What makes them so?

PROBLEM 1 Grill 'Em Up!

2

Your family is holding their annual cookout and you are in charge of buying food. On the menu are hamburgers and hot dogs. You have a budget determining how much you can spend. You have already purchased 3 packs of hot dogs at $2.29 a pack. You also need to buy the ground meat for the hamburgers. Ground meat sells for $2.99 per pound, but you are unsure of how many pounds to buy. You must determine the total cost of your shopping trip to know if you stayed within your budget.

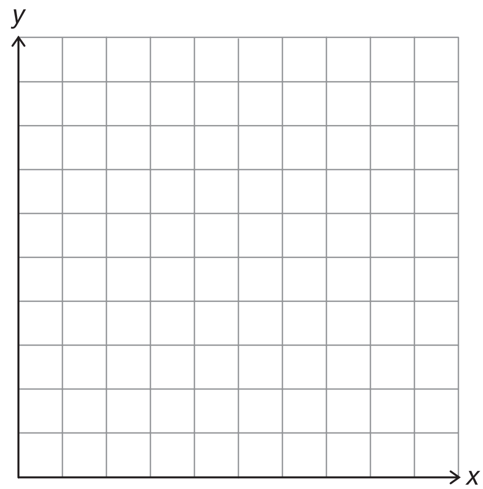
> This problem situation is represented by one of the following functions:
>
> $f(p) = 2.99p + 6.87$
>
> $f(p) = 2.29p^3 + 2.99p$
>
> $f(p) = |2.99p| + 6.87$
>
> $f(p) = 3p^2 + 2.29p + 2.99$

1. Choose a function to represent this problem situation. Explain your reasoning.

2. Complete the table to represent the total amount paid as a function of the amount of ground meat purchased. Don't forget to determine the units of measure.

	Independent Quantity	Dependent Quantity
Quantity		
Units		
Expression	p	
	0.5	
	1.75	
		13.60
		17.34
	4.25	

3. Use the data from the table to create a graph of the problem situation on the coordinate plane.



4. Consider a total bill of $13.45.

a. Estimate the amount of ground beef purchased.

b. Determine the exact amount of ground meat purchased.

5. Based on the number of people coming to the cookout, you decide to buy 6 pounds of ground meat for the hamburgers.

a. If your budget for the food is $25.00, do you have enough money? Why or why not?

b. If you have enough money, how much money do you have left over? If you do not have enough money, how much more will you need?

2

PROBLEM 2 Ground Breaking Costs

A construction company bought a new bulldozer for $125,000. The company estimates that its heavy equipment loses one-fifth of its value each year.

This problem situation is represented by one of the following functions:

$f(t) = 125{,}000t - \frac{1}{5}$

$f(t) = 125{,}000\left(\frac{4}{5}\right)^t$

$f(t) = \left|-\frac{1}{5}t \cdot 125{,}000\right|$

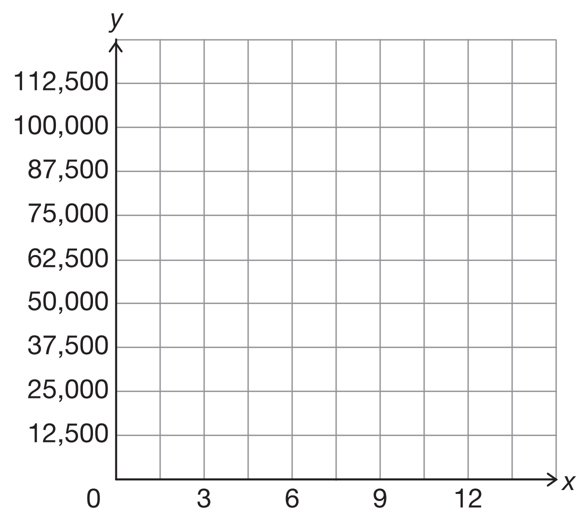
$f(t) = t^2 + 125{,}000t - \frac{1}{5}$

1. Choose a function to represent this problem situation. Explain your reasoning.

2. Complete the table to represent the cost of the bulldozer as a function of the number of years it is owned.

	Independent Quantity	Dependent Quantity
Quantity		
Units		
Expression	t	
	0	
	2.5	
	5	
	7	
	8.5	
	10	
	12.5	

3. Use the data and the function to graph the problem situation on the coordinate plane shown.



4. The owner wants to sell the bulldozer and make at least $25,000 in the sale.

   a. Estimate the amount of time the owner has to achieve this goal.

   b. Determine the exact amount of time the owner has to achieve this goal. Write your answer as an inequality.

5. When will the bulldozer be worth $0?

PROBLEM 3 Stick the Landing!

In gymnastics, it is important to have a mat below the equipment to absorb the impact when landing or falling. The thickness of the mats used in the rings, parallel bars, and vault events must be between 7.5 and 8.25 inches thick, with a target thickness of 7.875 inches.

This problem situation is represented by one of the following functions:

$f(t) = 7.875t - 0.375$

$f(t) = 7.875^t$

$f(t) = |t - 7.875|$

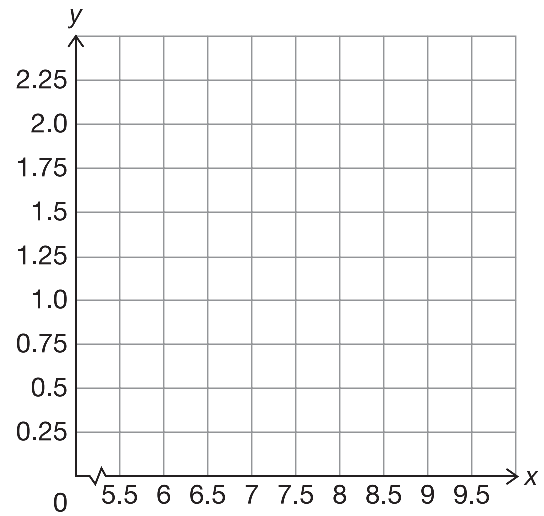
$f(t) = 7.875t^2 + 7.5t + 8.25$

1. Choose a function to represent this problem situation. Explain your reasoning.

2. Complete the table to represent the mat thickness in terms of the target thickness of the mat.

	Independent Variable	Dependent Variable
Quantity		
Units		
Expression	t	
	5.5	
	6.625	
		0.875
	7.5	
		0.25
		0.875
	9.25	
	9.875	

3. Use the data and the function to graph the problem situation on the coordinate plane shown.



4. The Olympics Committee announces that they will only use mats with a thickness of 7.875 inches and an acceptable difference of 0.375 inch.

   a. Write the absolute value inequality that represents this situation.

   b. Determine the thickest and thinnest mats that will be acceptable for competition. Write your solution as a compound inequality.

5. The All-Star Gymnastics Club has a practice mat with a thickness that is 1.625 inches off the Olympic recommendations. What are the possible thicknesses of the Gymnastics Club's practice mat?

PROBLEM 4 Fore!

In 1971, astronaut Alan Shepard hit a golf ball on the moon. He hit the ball at an angle of 45° with a speed of 100 feet per second. The acceleration of the ball due to the gravity on the moon is 5.3 feet per second squared. Then the ball landed.

This problem situation is represented by one of the following functions:

$f(d) = 5.3d$

$f(d) = 100^d + 5.3$

$f(d) = |5.3d| + 100$

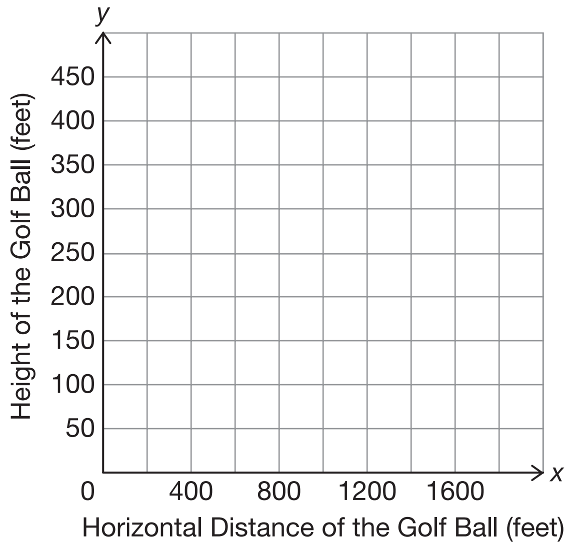
$f(d) = -\frac{5.3}{10,000}d^2 + d$

1. Choose a function to represent this problem situation.

2. Complete the table to represent the height of the golf ball in terms of the distance it was hit.

	Independent Quantity	Dependent Quantity
Quantity	Horizontal Distance of the Golf Ball	Height of the Golf Ball
Units		
Expression		
	405	
	745	
	945	
	1110	
	1335	
	1595	

3. Use the data and the function to graph the problem situation on the coordinate plane shown.

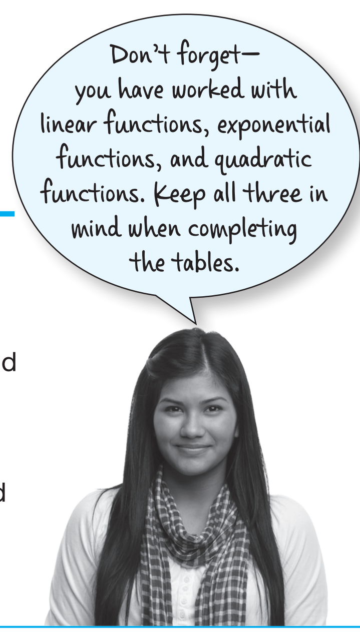


4. The Saturn V rocket that launched Alan Shepard into space was 363 feet tall. At what horizontal distance was the golf ball higher than the rocket was tall?

5. At what horizontal distance did the golf ball reach its maximum height? What was the greatest height the ball reached?

6. How far did the golf ball travel before it landed back on the moon?

Talk the Talk



In this chapter you used three different methods to determine values of various functions. You completed numeric tables of values, determined values from graphs, and solved equations algebraically. In addition, you used each of these methods by hand and with a
graphing calculator.

Think about each of the various methods for problem solving and complete the tables on the following pages. Pay attention to the unknown when describing each strategy.

	Numerically	
	Without Technology	**With Technology**
Given an Independent Quantity (input value)	**Description of the method:** **Advantages:** **Disadvantages/Limitation:**	**Description of the method:** **Advantages:** **Disadvantages:**
Given a Dependent Quantity (output value)	**Description of the method:** **Advantages:** **Disadvantages/Limitations:**	**Description of the method:** **Advantages:** **Disadvantages/Limitations:**

	Graphically	
	Without Technology	**With Technology**
Given an Independent Quantity (input value)	**Description of the method:** **Advantages:** **Disadvantages/Limitations:**	**Description of the method:** **Advantages:** **Disadvantages/Limitations:**
Given a Dependent Quantity (output value)	**Description of the method:** **Advantages:** **Disadvantages/Limitations:**	**Description of the method:** **Advantages:** **Disadvantages/Limitations:**

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	Algebraically	
	Without Technology	**With Technology**
Given an Independent Quantity (input value)	**Description of method:** **Advantages:** **Disadvantages/Limitations:**	**Description of method:** **Advantages:** **Disadvantages/Limitations:**
Given a Dependent Quantity (output value)	**Description of method:** **Advantages:** **Disadvantages/Limitations:**	**Description of method:** **Advantages:** **Disadvantages/Limitations:**

Be prepared to share your solutions and methods.

Chapter 2 Summary

KEY TERMS

- first differences (2.1)
- solution (2.1)
- intersection point (2.1)
- solve an inequality (2.3)
- compound inequality (2.4)
- solution of a compound inequality (2.4)
- conjunction (2.4)
- disjunction (2.4)
- opposites (2.5)
- absolute value (2.5)
- linear absolute value equation (2.5)
- linear absolute value inequality (2.5)
- equivalent compound inequality (2.5)

2

2.1 Identifying Dependent and Independent Quantities and Writing an Expression

The dependent quantity is dependent on how the independent quantity changes. The independent quantities are the input values of an expression and the dependent quantities are the output values.

Example

The table of values identifies the independent and dependent quantities and their units for the problem situation. An expression for the dependent quantity is written based on the independent quantity variable.

Caroline earns $25 a week babysitting after school. She deposits half of this amount in her savings account every Saturday.

	Independent Quantity	Dependent Quantity
Quantity	Time	Money Saved
Units	weeks	dollars
	0	0
	1	12.50
	2	25.00
	5	62.50
	10	125.00
Expression	w	$12.5w$

2.1 Determining the Unit Rate of Change

One way to determine the unit rate of change is to calculate first differences. First differences are calculated by taking the difference between successive points. Another way to determine the unit rate of change is to calculate the rate of change between any two ordered pairs and then write each rate with a denominator of 1. With two ordered pairs, the rate of change is the difference between the output values over the difference between the input values.

Example

Using first differences, the rate of change is 12.50.

	Time (weeks)	Money Saved (dollars)	First Differences
	0	0	
1 − 0 = 1			12.50 − 0 = 12.50
	1	12.50	
2 − 1 = 1			25.00 − 12.50 = 12.50
	2	25.00	

Using two ordered pairs, the rate of change is 12.50.

(2, 25.00) and (5, 62.50) $\frac{62.50 - 25.00}{5 - 2} = \frac{37.50}{3} = \frac{12.50}{1}$

2.1 Determining the Solution to a Linear Equation Using Function Notation

To write a linear equation in function notation, $f(x) = ax + b$, identify the dependent (output value) and independent (input value) quantities and the rate of change in a problem situation. Determine a solution to the equation by substituting a value for the independent quantity in the equation.

Example

Caroline earns $25 a week babysitting after school. She deposits half of this amount in her savings account every Saturday.

$s(w) = 12.5w$

$s(14) = 12.5(14)$ Caroline will have $175 saved after 14 weeks.

$s(14) = 175$

2.1 Determining the Solution to a Linear Equation on a Graph Using an Intersection Point

A graph can be used to determine an input value given an output value. The graph of any function, f, is the graph of the equation $y = f(x)$. On the graph of any equation, the solution is any point on that line. If there are intersecting lines on the graph, the solution is the ordered pair that satisfies both equations at the same time, or the intersection point of the graph. To solve an equation using a graph, first graph each side of the equation and then determine the intersection point.

Example

Caroline earns $25 a week babysitting after school. She deposits half of this amount in her savings account every Saturday. How long will it take Caroline to save $300?

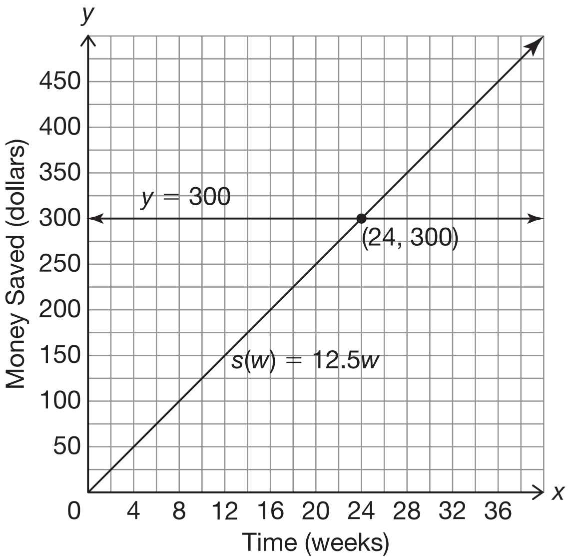
$s(w) = 12.5w$

$300 = 12.5w$

$y = 300 \qquad y = 12.5x$

The solution is (24, 300).

It will take Caroline 24 weeks to save $300.



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2.2 Identifying and Describing the Parts of a Linear Function

Identifying each expression in a linear function, its units, its meaning in terms of the problem situation, and its mathematical meaning can help you determine the solution for a linear function. The independent quantity is the input value and the dependent quantity is the output value. The y-intercept is the point on the graph where x equals 0.

Example

Tyler has $100 in his car fund. He earns $7.50 per hour at his after-school job. He works 3 hours each day, including weekends. Tyler saves 100% of his earned money in his car fund.

		Description	
Expression	**Unit**	**Contextual Meaning**	**Mathematical Meaning**
d	day	the time, in days, that the money has been saved	input value
22.50	$\frac{\text{dollars}}{\text{days}}$	the amount of money that is saved each day	rate of change
$22.50d$	dollars	money saved to car fund	
100	dollars	the amount of money already in car fund	y-intercept
$100 + 22.50d$	dollars	the total amount of money in car fund	output value

2.2 Comparing Tables, Equations, and Graphs to Model and Solve Linear Situations

A table can help you calculate solutions given a few specific input values. A graph can help you determine exact solutions if the graph of the function crosses the grid lines exactly. A function can be solved for any value, so any and all solutions can be determined. A graphing calculator allows for more accuracy when using a graph to determine a solution.

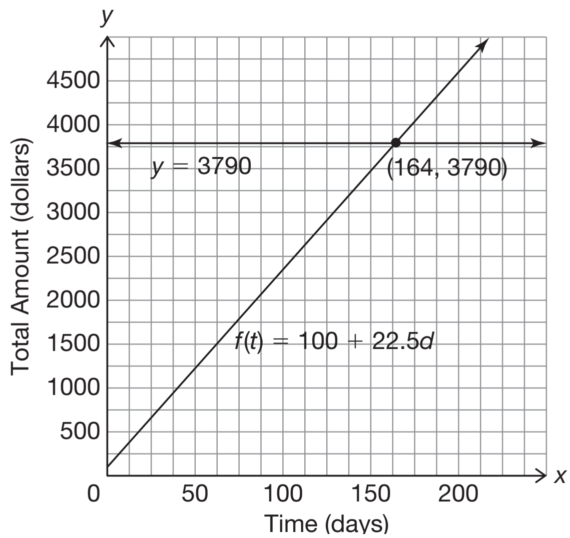
2

Example

Tyler had \$100 in his car fund. He earns \$7.50 per hour at his after-school job. He works 3 hours each day, including weekends. Tyler saves 100% of his earned money in his car fund. How many days will it take him to have enough money to buy a car that costs \$3790?

A table can be used to estimate that it will take between 100 and 175 days to buy the car. A graph can be used to estimate that it will take about 160 days to buy the car. A function will give an exact solution. It will take exactly 164 days to buy a car that costs \$3790.

t	100 + 22.50*t*
0	100
10	325
20	550
50	1225
100	2350
175	4037.5



$$f(t) = 100 + 22.50d$$

$$3790 = 100 + 22.50d$$

$$3690 = 22.50d$$

$$\frac{3690}{22.50} = \frac{22.50d}{22.50}$$

$$164 = t$$

2.3 Writing and Solving Inequalities

When solving an inequality, first write a function to represent the problem situation. Then write the function as an inequality based on the independent quantity. To solve an inequality, determine the values of the variable that make the inequality true. The objective when solving an inequality is similar to the objective when solving an equation: Isolate the variable on one side of the inequality symbol. Finally, interpret the meaning of the solution.

Example

Cameron has $25 in his gift fund which he is going to use to buy his friends gifts for graduation. Graduation is 9 weeks away. If he would like to have at least $70 to buy gifts for his friends, how much should he save each week?

The function is $f(x) = 25 + 9x$, so the inequality would be $25 + 9x \geq 70$.

$$25 + 9x \geq 70$$
$$9x \geq 45$$
$$\frac{9x}{9} \geq \frac{45}{9}$$
$$x \geq 5$$

Cameron would need to save at least $5 each week to meet his goal.

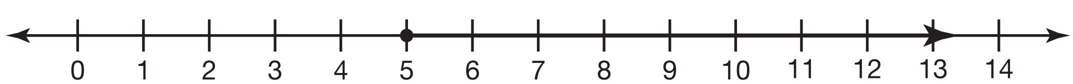
2.3 Representing Inequalities on a Number Line

A number line can be used to represent the solution of an inequality. After solving the inequality, draw a point on the number line at the value of the solution. The point should be closed if the value is included in the solution and open if the value is not included. An arrow should be drawn to the right if the solution is greater than and to the left if the solution is less than.

Example

Cameron has $25 in his gift fund which he is going to use to buy his friends gifts for graduation. Graduation is 9 weeks away. If he would like to have at least $70 to buy gifts for his friends, how much should he save each week?

The function is $f(x) = 25 + 9x$ and the inequality would be $25 + 9x \geq 70$. Cameron needs to save at least $5 each week. When $f(x) = 70$, $x = 5$.



2.3 Representing Inequalities on a Coordinate Plane

Inequalities can be represented on a coordinate plane by first graphing the linear function related to the inequality. A point is drawn representing the solution of the inequality. A dashed box can be used to represent the area of the solution that is less than the quantity and an oval can be used to represent the section of the solution that is greater than the quantity.

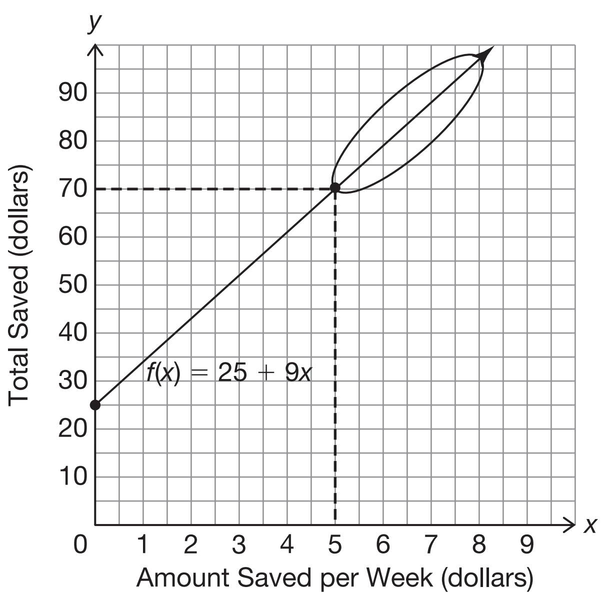
Example

Cameron has $25 in his gift fund which he is going to use to buy his friends gifts for graduation. Graduation is 9 weeks away. If he would like to have at least $70 to buy gifts for his friends, how much should he save each week?

2

The function is $f(x) = 25 + 9x$, so the inequality would be $25 + 9x \geq 70$.

The point at (5, 70) means that at $5 saved per week, the total savings is equal to $70. The box represents all of the amounts saved per week, x, that would leave Cameron with less than $70 saved by graduation. The oval represents all of the amounts saved per week, x, that would leave Cameron with $70 or more saved by graduation.



2.3 Solving an Inequality with a Negative Rate of Change

When you divide each side of an inequality by a negative number, the inequality sign reverses.

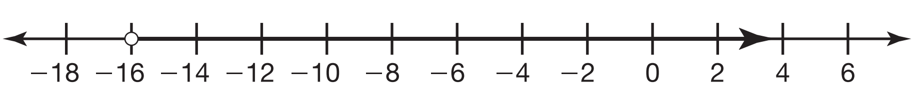
Example

$$250 - 9.25x < 398$$

$$-9.25x < 148$$

$$\frac{-9.25x}{-9.25} > \frac{148}{-9.25}$$

$$x > -16$$



2.4 Writing Compound Inequalities

A compound inequality is an inequality that is formed by the union, "or," or the intersection, "and," of two simple inequalities. Compound inequalities containing "and" can be written in compact form.

Example

You pay a discounted rate if you are 12 years of age or less or 65 years of age or more.

$x < 12$ or $x > 65$

You will pay the full rate if you are more than 12 years of age and less than 65 years of age.

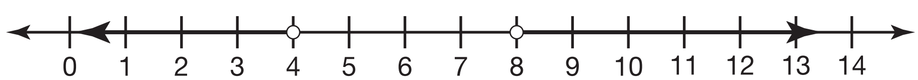
$x > 12$ and $x < 65$; $12 < x < 65$

2.4 Representing the Solutions to Compound Inequalities on a Number Line

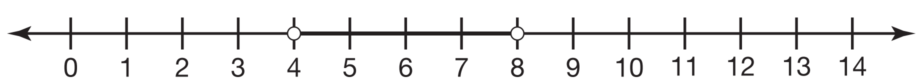
The solution of a compound inequality in the form $a < x < b$, where a and b are any real numbers, is the part or parts of the solutions that satisfy both of the inequalities. This type of compound inequality is called a conjunction. The solution of a compound inequality in the form $x < a$ or $x > b$, where a and b are any real numbers, is the part or parts of the solution that satisfy either inequality. This type of compound inequality is called a disjunction. Graphically, the solution to a disjunction is all the regions that satisfy the separate inequalities. Graphically, the solution to a conjunction is the intersection of the separate inequalities.

Example

$x < 4$ or $x > 8$



$4 < x < 8$

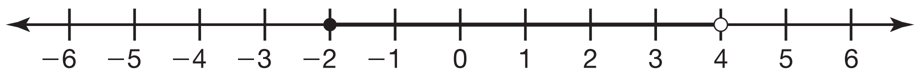


2.4 Solving Compound Inequalities

To solve a compound inequality written in compact form, isolate the variable between the two inequality signs, and then graph the resulting statement. To solve an inequality involving "or," simply solve each inequality separately, keeping the word "or" between them, and then graph the resulting statements.

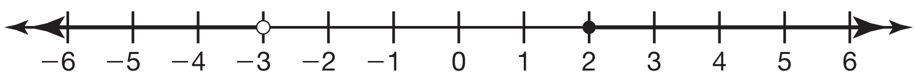
Example

a.



$$\begin{aligned} -3 &\le 4x + 5 < 21 \\ -3 - 5 &\le 4x + 5 - 5 < 21 - 5 \\ -8 &\le 4x < 16 \\ \frac{-8}{4} &\le \frac{4x}{4} < \frac{16}{4} \\ -2 &\le x < 4 \end{aligned}$$

b.



$$\begin{aligned} 3 - 2x &> 9 \\ 3 - 3 - 2x &> 9 - 3 \\ -2x &> 6 \\ \frac{-2x}{-2} &< \frac{6}{-2} \\ x &< -3 \end{aligned}$$

or

$$\begin{aligned} 4x + 1 &\ge 9 \\ 4x + 1 - 1 &\ge 9 - 1 \\ 4x &\ge 8 \\ \frac{4x}{4} &\ge \frac{8}{4} \\ x &\ge 2 \end{aligned}$$

2.5 Solving Linear Absolute Value Equations

To solve linear absolute value equations, write both the positive and negative equations that the linear absolute value equation represents. Then solve each equation.

Example

$$\begin{aligned} |5x - 4| &= 21 \\ +(5x - 4) &= 21 \\ 5x - 4 &= 21 \\ 5x - 4 + 4 &= 21 + 4 \\ 5x &= 25 \\ \frac{5x}{5} &= \frac{25}{5} \\ x &= 5 \end{aligned}$$

$$\begin{aligned} -(5x - 4) &= 21 \\ \frac{-(5x - 4)}{-1} &= \frac{21}{-1} \\ 5x - 4 &= -21 \\ 5x - 4 + 4 &= -21 + 4 \\ 5x &= -17 \\ \frac{5x}{5} &= \frac{-17}{5} \\ x &= -3\frac{2}{5} \end{aligned}$$

2.5 Writing and Evaluating Linear Absolute Value Inequalities

If there is a range of solutions that satisfy a problem situation, you can write an absolute value inequality. To evaluate for a specific value, substitute the value for the variable.

Example

A swimmer who wants to compete on the green team at the City Swim Club should be able to swim the 100-meter freestyle in 54.24 seconds plus or minus 1.43 seconds. Can a swimmer with a time of 53.15 seconds qualify for the green team?

$$|t - 54.24| \le 1.43$$

$$|53.15 - 54.24| \le 1.43$$

$$|-1.09| \le 1.43$$

$$1.09 \le 1.43$$

The swimmer qualifies because his time is less than 1.43 seconds from the base time.

2.5 Representing Linear Absolute Value Inequality Solutions Graphically

All values within the solution to a linear absolute value inequality can be represented along a number line or on a coordinate plane. A box and an oval can be used to identify values greater or less than the solution.

Example

$$|t - 54.24| \le 1.43$$

$$+(t - 54.24) \le 1.43$$

$$t - 54.24 \le 1.43$$

$$t \le 55.67$$

$$-(t - 54.24) \le 1.43$$

$$\frac{-(t - 54.24)}{-1} \le \frac{1.43}{-1}$$

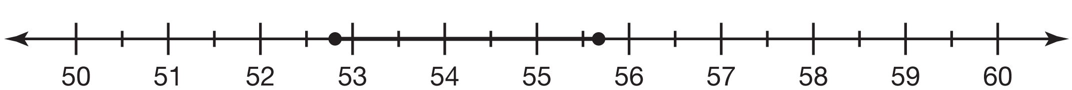
$$t - 54.24 \ge -1.43$$

$$t - 54.24 + 54.24 \ge -1.43 + 54.24$$

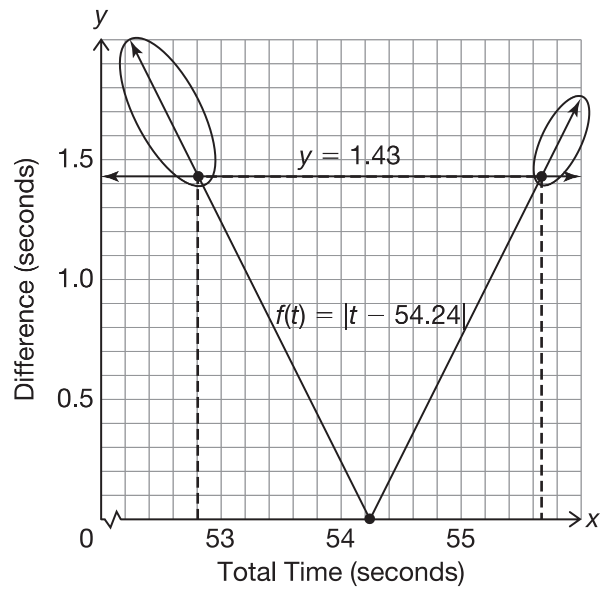
$$t \ge 52.81$$

The solution is $52.81 \le t \le 55.67$.

Solution:



Graph of $f(t) = |t - 54.24|$:

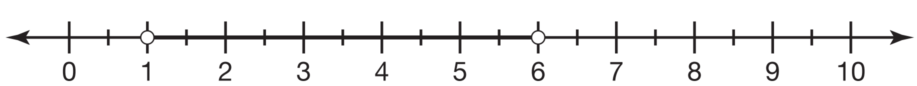


2.5 Solving and Graphing Linear Absolute Value Inequalities on a Number Line

Absolute value inequalities can take four different forms with the absolute value expression compared to a value, c. To solve an absolute value inequality, you must first write it as an equivalent compound inequality. "Less than" inequalities will be conjunctions and "greater than" inequalities will be disjunctions.

Example

$$4 + |2x - 7| < 9$$
$$4 - 4 + |2x - 7| < 9 - 4$$
$$|2x - 7| < 5$$
$$-5 < 2x - 7 < 5$$
$$-5 + 7 < 2x - 7 + 7 < 5 + 7$$
$$2 < 2x < 12$$
$$\frac{2}{2} < \frac{2x}{2} < \frac{12}{2}$$
$$1 < x < 6$$



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2.6 Determining Solutions for Nonlinear Functions Graphically Using Intersection Points

Graphs can be used to determine solutions for linear or non-linear functions. First, graph each side of the inequality on the coordinate plane, and then locate and label the intersection point. The box and oval method can be used to identify the solution to a non-linear inequality.

Example

Jonah bought a rare collectible for $150 that is supposed to gain one-fifth of its value each year. He wants to wait to sell the collectible until it's worth at least $500.

$f(t) = 150(1.2)^t$

$150(1.2)^t \geq 500$

The collectible will be worth $500 after about 6.6 years. Jonah could sell the collectible any time after 6.6 years and it will be worth at least $500.