Linear Functions

3

While flowers may brighten an office building lobby, can accompany a trophy at an awards ceremony, or are centerpieces on dinner tables, there is a lot more than roses and tulips—there's mathematics too. This is especially true if flower vendors are considering how to promote their businesses.

161

Is It Getting Hot in Here?

Modeling Data Using Linear Regression

LEARNING GOALS

In this lesson, you will:

- Create a graph of data points on a graphing calculator.
- Determine a linear regression equation using a graphing calculator.
- Recognize the accuracy of a line of best fit using the correlation coefficient.
- Make predictions about data using a linear regression equation.

KEY TERMS

- linear regression
- line of best fit
- linear regression equation
- significant digits
- correlation coefficient

You may have heard a lot of conversation recently about global warming. But did you know that just 40 years ago, some scientists thought the Earth was actually going through a time of global cooling?

In the 1970s, scientists became aware that the averages of global temperatures had decreased since the 1940s. Evidence shared at a climate conference in 1965 triggered thought that changes in sunlight might cause an ice age which would begin within a few thousand years. At that time, much less was known about world climate and a few severe winters in 1972 and 1973 had the public believing this theory.

These thoughts and theories were all based on data which was then used to predict trends. Do you think this is a reliable method? Why might scientists use this method, even if it is not always correct?

What's Your Prediction?

The table shown lists the average global temperature for each decade from 1880 to 2009.

Decade Number	Decade	Average Temperature (°F)
0	1880–1889	56.876
1	1890–1899	56.642
2	1900–1909	56.732
3	1910–1919	56.822
4	1920–1929	57.038
5	1930–1939	57.236
6	1940–1949	57.290
7	1950–1959	57.164
8	1960–1969	57.092
9	1970–1979	57.236
10	1980–1989	57.668
11	1990–1999	57.920
12	2000–2009	58.316

1. Identify the independent and dependent quantities and their units of measure.

2. Do the data represent a function? Why or why not? If so, describe the function.

You can represent the data using a graphing calculator. In order to enter the data in your calculator, you must represent each decade as a single value.

3. Follow the steps provided to graph the relationship between time and temperature on a graphing calculator.

You can use a graphing calculator to represent a data set.

Step 1: Press **STAT** and then press **ENTER** to select **1:Edit**. In the **L1** column, enter the independent quantity values by typing each value followed by **ENTER**.

Step 2: Use the right arrow key to move to the **L2** column. Enter the dependent quantity values.

Step 3: Press **2ND** and **STAT PLOT**. Press **4** to turn off any plots. Press **ENTER**. Then press **2ND** and **STAT PLOT** again. Press **ENTER** to access the information about Plot 1. The cursor should be on the word **On**. Press **ENTER** to turn on Plot 1.

> If there is already data in your L1 list, highlight the heading L1, Press CLEAR, then Press ENTER to delete it.

> Remember to use the decade number for the independent quantity and not the actual decade years.

Step 4: Use the arrow keys to move down to **Xlist**. Press **2ND L1** to set your L1 values as your x-values. Scroll to **Ylist** and Press **2ND L2** to set your L2 values as your y-values.

Step 5: Press **WINDOW** to set the bounds of your graph. Press **GRAPH** to create a graph of the data.

Step 6: Use the **TRACE** feature and the left and right arrow keys to move between the points on the plot.

4. Why do you think the first decade is numbered 0?

5. Between which consecutive decades was there a decrease in average global temperature?

6. What is the range of the data set?

7. Is it possible to predict the approximate average global temperature for 2070–2079 using the graph? Explain your reasoning.

8. Would it make sense to draw a smooth curve connecting the points in the plot? Why or why not?

Scientists often use a *linear regression* to model data in order to make predictions. A **linear regression** models the relationship between two variables in a data set by producing a *line of best fit*. A **line of best fit** is the line that best approximates the linear relationship between two variables in a data set. The equation that describes the line of best fit is called the **linear regression equation**.

You can use a graphing calculator to determine a linear regression equation and then draw a line of best fit for the average global temperature data.

> You can use a graphing calculator to determine the linear regression equation of a data set.
>
> **Step 1:** **Diagnostics** must be turned on so that all needed data is displayed. Press **2nd CATALOG** to display the catalog. Scroll to **DiagnosticOn** and press **ENTER**. Then press **ENTER** again. The calculator should display the word **Done**.
>
> **Step 2:** Press **STAT** and use the right arrow key to show the **CALC** menu. Type **4** to choose **LinReg(ax+b)** and press **ENTER**.
>
> **Step 3:** Make sure **L1** is listed next to **Xlist** and **L2** is listed next to **Ylist**. Scroll down to **Calculate** and press **ENTER**.
>
> The calculator should show $y = ax + b$ as well as four values labeled **a, b, r²,** and **r**.

> You will need to substitute the values of *a* and *b* into the equation to write the linear regression equation.

1. Determine the linear regression equation for the average global temperature data.

Your graphing calculator may provide numbers rounded to several decimal places for the values used in the linear regression equation. However, are all those decimal places necessary? Using *significant digits* can help you make more sense of the values. **Significant digits** are digits that carry meaning contributing to a number's precision.

In this problem situation, the temperature measurements are accurate to the thousandths place, so each temperature measurement has 3 significant decimal digits. When constructing equations and expressions based on data, it is appropriate to use numbers that have the same number of significant decimal digits.

2. Rewrite the linear regression equation as a function. This time, round the slope and *y*-intercept to the appropriate place. Explain your reasoning.

Let's graph this linear regression equation to see if it is an appropriate fit for the data.

> **Step 4:** Press **Y=**. Enter the linear regression equation next to **Y₁=**. Then press **GRAPH** to see the line of best fit.
>
> The calculator will automatically copy the linear regression equation to Y1 if you enter LinReg(*ax + b*) **Y1**. Repeat Step 2 to enter LinReg(*ax + b*), then press **VARS** and use the right arrow keys to show **Y-VARS**. Press **FUNCTION** and select **Y1**. When you press **ENTER** the equation will appear in **Y1**.

3. Sketch the data points and the line of best fit that you see on the calculator.
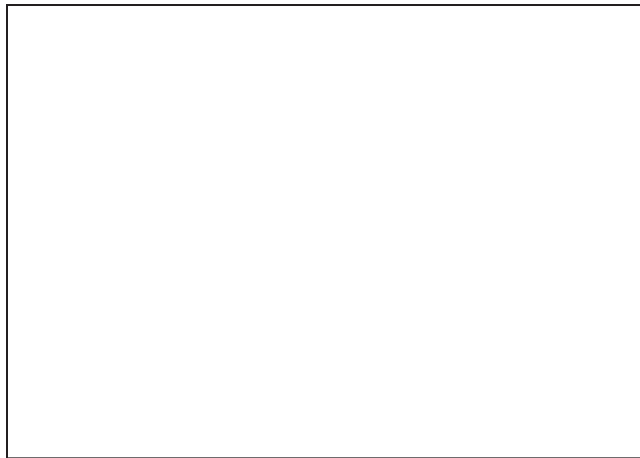
a. Do the data show a positive correlation or a negative correlation? How can you tell?

b. Do you think this line fits the data well? Explain your reasoning.

The variable r on your linear regression screen is used to represent the *correlation coefficient*. The **correlation coefficient** indicates how closely the data points form a straight line.

If the data show a positive correlation, then the value of r is between 0 and 1. The closer the data are to forming a straight line, the closer the value of r is to 1.

If the data show a negative correlation, the value of r is between 0 and -1. The closer the data are to forming a straight line, the closer the r-value is to -1.

If there is no linear relationship in the data, the value of r is 0.

You will also see an r^2 value on your screen. That is called the coefficient of determination. We will get to that in a later chapter.

4. What is the correlation coefficient, or r-value, for your line of best fit? Interpret the meaning of the r-value.

5. Compare your sketch with your classmates' sketches. Locate a classmate who used bounds different from your bounds. How does changing the **Xmin** and **Xmax** change the look of the graph? How does changing the **Ymin** and **Ymax** change the look?

1. For each expression from your linear regression equation, write an appropriate unit of measure and describe the contextual meaning. Then, choose a term from the word box to describe the mathematical meaning of each part.

output value   rate of change   input value   *y*-intercept

Expression	Unit	What It Means	
		Contextual Meaning	**Mathematical Meaning**
f(x)			
0.11			
x			
56.572			

Use your linear regression equation to answer each question.

2. About how much was the average global temperature changing each decade from 1880 to 2009 according to the data? Explain how you know.

3. Compare the *y*-intercept from the table with the *y*-intercept from the linear regression equation. What do you notice? Does this make sense in terms of the problem situation? Why or why not?

4. Use your equation to predict the average global temperature for the years 2070–2079. Show your work and explain your reasoning.

5. Predict the first decade with an average global temperature of at least 60°F.

Be prepared to share your solutions and methods.

3

Tickets for Sale
Standard Form of Linear Equations

LEARNING GOALS

In this lesson, you will:

- Identify contextual meaning of expressions in an function.
- Write equations in standard form.
- Solve equations in standard form.
- Determine the x-intercept and y-intercept of an equation in standard form.
- Use intercepts to graph an equation.
- Convert equations from standard form to slope-intercept form.
- Solve equations in slope-intercept form.
- Determine the x-intercept and y-intercept of an equation in slope-intercept form.
- Perform unit analysis of equations.

KEY TERMS

- standard form
- slope-intercept form

Have you ever wondered why many prices end in 0.99 instead of a multiple of 5 or 0? There are many theories on when and how this trend started. One theory is that in 1876 Melville E. Stone wanted to sell his newspaper for 1¢ to compete against other 5¢ newspapers. The problem was, not many people used pennies in those days. Stone convinced shopkeepers to set their prices to end in $0.99 so consumers would get the penny back as change. People then used their pennies to buy his newspapers and the pricing stuck. Another theory is that stores starting slightly lowering their prices to compete with other stores. Wouldn't you rather buy something that costs $2.99 at one store instead of $3.00 at another? One final theory is called psychological pricing. This states that consumers tend to ignore the least significant digits rather than do the proper rounding. We may see $19.99 as $19 instead of closer to $20, which is correct.

Do you agree with any of these theories? Can you think of any other reasons stores may set their prices to end in these amounts?

The Marshall High School Athletic Association sells tickets for the weekly football games. Students pay $5 and adults pay $10 for a ticket.

1. How much money would the athletic association collect:

   a. if 100 students and 50 adults buy tickets to the game?

   b. if 125 students and 75 adults buy tickets to the game?

3

   c. if 97 students and 116 adults buy tickets to the game?

2. Explain how you can determine the total amount of money collected if you know the number of student tickets sold and the number of adult tickets sold.

3. Write an expression to represent the total amount of money collected for any number of tickets sold. Use *s* to represent the number of student tickets sold, and *a* to represent the number of adult tickets sold.

4. Explain how your expression represents this problem situation.

The expression $5s + 10a$ can be written in function notation as a function of two variables.

$$f(s, a) = 5s + 10a$$

You can use this function to determine the amount of money collected if 75 student tickets and 45 adult tickets are sold.

$$f(75, 45) = 5(75) + 10(45)$$

$$f(75, 45) = 375 + 450$$

$$f(75, 45) = 825$$

The athletic association collected \$825 if they sold 75 student tickets and 45 adult tickets.

> Do I always have to write it as $f(s, a) = 5s + 10a$? Could I write it as $f(s, a) = 10a + 5s$? Or $f(a, s) = 5s + 10a$? Does the order matter?

5. What unit of measure describes $f(s, a)$? Explain your reasoning.

6. Use the function $f(s, a)$ to determine how much money is collected at each game.

   a. Game 1

      $f(85, 80)$

   b. Game 2

      $f(94, 63)$

   c. Game 3

      $f(54, 72)$

   d. Game 4

      $f(123, 108)$

7. If you know the number of student tickets sold, can you determine the total amount of money collected? Explain your reasoning.

8. If you know the total amount of money collected, can you determine the number of student and adult tickets sold? Explain your reasoning.

PROBLEM 2 How Many Tickets Do We Need to Sell?

The football team is playing in an out-of-town tournament. The athletic association needs to raise $3000 to send the team to this tournament. The money raised from selling tickets to a special event home game will be used toward the tournament cost.

1. Write an equation to represent this problem situation using the same prices from Problem 1.

2. How does this equation differ from the function you wrote in Problem 1?

3. Use your equation to complete the table.

Quantity Name					
Unit					
Expression	s	a	$5s$	$10a$	$5s + 10a$
	250				
		150			
				3000	
	600				

4. Carla and Robena sell game tickets. They have already sold 95 student tickets. Carla says that they need to sell 252 adult tickets to reach the goal of $3000. Robena says that they need to sell 253 adult tickets to reach the goal. Who is correct? Explain your reasoning.

PROBLEM 3　Analyzing Ticket Sales

Students that want to attend the special event game must purchase their tickets at school prior to the game. So far, 189 students bought tickets for the game. The athletic association wants to know how many adult tickets they must sell in order to reach their goal of $3000. However, they want a method to make forecasting how many adult tickets they must sell more efficient.

Another way to determine the number of adult tickets that must be sold to reach a goal of $3000 is to transform the equation to isolate a first.

$$5s + 10a = 3000$$

$$5s - 5s + 10a = -5s + 3000$$

$$\frac{10a}{10} = \frac{-5s}{10} + \frac{3000}{10}$$

$$a = -\frac{1}{2}s + 300$$

Now, substitute the information you know into the transformed equation.

This form of the equation looks really familiar . . .

1. How many adult tickets must be sold to reach the goal of $3000?

Now that we have transformed the original equation, where did the $3000 goal part go?

2. Let the x-axis represent the number of student tickets sold. Let the y-axis represent the number of adult tickets sold.

   a. Determine the x-intercept and the y-intercept for the transformed equation. Explain what each intercept means in terms of the problem situation. What do you notice?

3. Use the *x*-intercept and *y*-intercept to graph the equation.
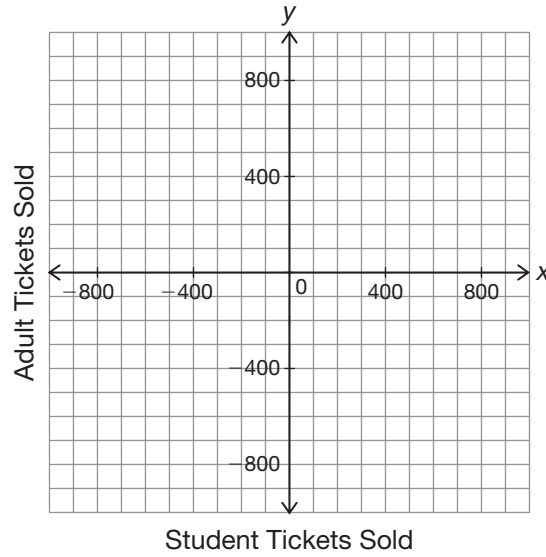
4. Identify the slope of the graph. Interpret its meaning in terms of the problem situation.

5. The athletic association sold 400 student tickets. Determine how many adult tickets they must sell to reach the $3000 goal.

6. Can you use the graph to determine how much money is collected if the athletic association sold 400 student tickets and 200 adult tickets? Why or why not?

7. Let's consider reaching the $3000 goal for ticket sales by analyzing the number of adult tickets sold. If the association knows that 150 adult tickets have been sold, how many student tickets would they need to sell to reach their goal?

   a. Transform the equation $5s + 10a = 3000$ to solve for the number of student tickets.

> Remember, they have already sold the maximum number of adult tickets!

   b. How many student tickets must the athletic association sell on homecoming weekend to reach their goal of $3000?

   c. Determine the *x*-intercept and the *y*-intercept of the graph described by this equation. Explain what the intercepts mean in terms of the problem situation.

8. Use the *x*-intercept and the *y*-intercept to graph the equation.
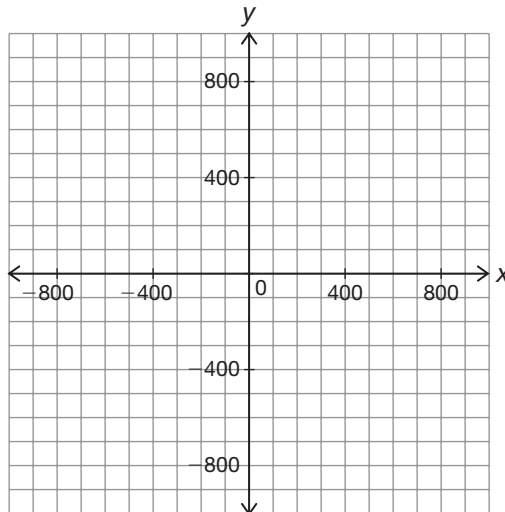
> Remember to label your axes before graphing!

9. Identify the slope of the graph. Interpret its meaning in terms of the problem situation.

10. Compare the *x*-intercepts and the *y*-intercepts of the two graphs you just created. What do you notice?

11. Is there a way to determine the total amount of money collected from either graph? Explain why or why not.

You may have noticed that when you transformed the equations to isolate the *a* or *s* variables, the $3000 goal "disappeared." Where did the money go? Let's perform unit analysis on each part of one isolated *a* equation to see just where that $3000 went.

> Remember, when you're doing unit analysis, you have to identify the units for *each* part of the equation!

12. Identify the units of measure for each part of the equation, $5s + 10a = 3000$.

    a. 5   **b.** *s*

    c. 10   **d.** *a*

    e. 3000

When you analyze the units of each part of the equation, you can see how the equation maintains balance.

$$5s + 10a = 3000$$

| 5 | · | s | + | 10 | · | a | = | 3000 |

$$\frac{\text{dollars}}{\text{student tickets}} \cdot \text{student tickets} + \frac{\text{dollars}}{\text{adult tickets}} \cdot \text{adult tickets} = \text{dollars}$$

$$\frac{\text{dollars}}{\cancel{\text{student tickets}}} \cdot \cancel{\text{student tickets}} + \frac{\text{dollars}}{\cancel{\text{adult tickets}}} \cdot \cancel{\text{adult tickets}} = \text{dollars}$$

13. Write the next sentence in the worked example after dividing out the two different units of measure. What does this tell you about the original equation?

When you rewrote the equation to determine the number of adult tickets sold, the units were rearranged.

When you transformed the equation to determine the number of adult tickets sold, the units were also rearranged.

$$5s + 10a = 3000$$

$$10a = -5s + 3000$$

$$\frac{10a}{10} = \frac{-5s}{10} + \frac{3000}{10}$$

$$a = -\frac{1}{2}s + 300$$

Let's consider the units of each part of the highlighted equation to show how they simplify:

$$\frac{10a}{10} \quad \text{simplifies to} \quad a$$

$$\frac{10 \cdot a}{10}$$

$$\frac{\dfrac{\text{dollars}}{\text{adult tickets}} \cdot \text{adult tickets}}{\dfrac{\text{dollars}}{\text{adult tickets}}} \qquad \frac{\dfrac{\text{dollars}}{\text{dollars}}}{\text{adult tickets}} = \text{dollars} \cdot \frac{\text{adult tickets}}{\text{dollars}} \qquad \text{adult tickets}$$

$$\frac{-5s}{10} \quad \text{simplifies to} \quad -\frac{1}{2}s$$

$$\frac{-5 \cdot s}{10}$$

$$\frac{\dfrac{\text{dollars}}{\text{student tickets}} \cdot \text{student tickets}}{\dfrac{\text{dollars}}{\text{adult tickets}}} \qquad \frac{\dfrac{\text{dollars}}{\text{dollars}}}{\text{adult tickets}} = \text{dollars} \cdot \frac{\text{adult tickets}}{\text{dollars}} \qquad \text{adult tickets}$$

$$\frac{3000}{10} \quad \text{simplifies to} \quad 300$$

$$\frac{3000}{10}$$

$$\frac{\dfrac{\text{dollars}}{\text{dollars}}}{\text{adult tickets}} \qquad \text{dollars} \cdot \frac{\text{adult tickets}}{\text{dollars}} \qquad \text{adult tickets}$$

So, in $a = \frac{1}{2}s + 300$, all the units are balanced.

adult tickets = adult tickets + adult tickets

Linear functions can be written in two forms, *standard form* and *slope-intercept* form.

The **standard form** of a linear equation is $Ax + By = C$ where A, B, and C are constants and A and B are not both zero.

In the first worked example, the original equation was written in standard form with the dollars units already isolated on one side of the equation.

The **slope-intercept form** of a linear equation is $y = mx + b$ where b is the y-intercept and m is the slope.

To isolate the a-variable in the second worked example, you transformed the original equation written in standard form to the slope-intercept form with the a-variable representing the dependent quantity.

As you have seen in this problem situation, it is often necessary to change between the forms as one form might be more appropriate to represent a situation.

14. Explain what happened to the units of students and dollars when converting from standard form to slope-intercept form in the worked example.

15. Convert the standard form of the original equation $5s + 10a = 3000$ to slope intercept form to represent the number of student tickets. Show and explain the final units for the equation.

Talk the Talk

Katie received a $75 gift card for her birthday. She decides to buy new music and movies for her electronic notebook with the gift card. Songs cost $1.29 each and movies cost $14.99 each.

1. Write an equation to represent this problem situation. Use s to represent the number of songs and m to represent the number of movies.

2. Complete the table to show what each expression represents in this problem situation.

Expression	Contextual Meaning
s	
m	
1.29	
14.99	
$1.29s$	
$14.99m$	
$1.29s + 14.99m$	
75	

3. If Katie buys 20 songs, what is the greatest number of movies she can buy?

4. If Katie buys no movies, what is the greatest number of songs she can buy? What does this number represent?

5. If Katie buys no songs, what is the greatest number of movies she can buy? What does this number represent?

Be prepared to share your solutions and methods.

Cool As A Cucumber or Hot Like A Tamale!

Literal Equations in Standard Form and Slope-Intercept Form

LEARNING GOALS

In this lesson, you will:

- Recognize and use literal equations.
- Convert literal equations to highlight a specific variable.
- Convert between standard and slope-intercept form.
- Recognize the value of standard and slope-intercept form.

KEY TERM

- literal equation

Converting between degrees Celsius and degrees Fahrenheit is knowledge that you might need, especially if you travel or live outside of the United States. The formula $C = \frac{5}{9}(F - 32)$ can seem confusing and difficult to remember, especially if you don't understand where the numbers are coming from. So what is so special about $\frac{5}{9}$? Why is −32 so important to the formula?

The freezing point of water on the Fahrenheit scale is 32°. The freezing point for water on the Celsius scale is 0°. This 32 degree difference is why −32 is used in the formula. As for the fraction $\frac{5}{9}$, the difference between the freezing point of water and the boiling point of water in degrees Celsius is 100°. That same difference in degrees Fahrenheit is equal to 180°. This means the scales rise at different rates. This change in rates of $\frac{100}{180}$ can be simplified to $\frac{5}{9}$. Of course, when you convert the formula to solve for Fahrenheit, these numbers change slightly, but you should still be able to understand where they come from.

Want to make this formula even easier? The decimal form of $\frac{5}{9}$ is equivalent to about 0.56, which is just about half. Subtract the degrees Fahrenheit by 30 and then divide that difference by 2. It's not exact, but it should give you a pretty close estimate. You should still be able to tell whether you should wear a winter coat or a tank top!

187

Is It Hot in Here?

Literal equations are equations in which the variables represent specific measures. You most often see literal equations when you study formulas. These literal equations can be manipulated in order to allow you to solve for one specific variable.

A common literal equation is the formula for converting degrees Fahrenheit to degrees Celsius.

$$C = \frac{5}{9}(F - 32)$$

3

1. The normal temperature for the human body is 98.6°F. What temperature is that in degrees Celsius?

2. The coldest temperature ever recorded on Earth was in Antarctica in 1983. The temperature recorded was −126.8°F. What is this temperature in degrees Celsius?

3. The directions on a box of cake batter tells you to bake your cake at 177°C. What is this temperature in degrees Fahrenheit?

4. Is there a more efficient way to determine degrees Fahrenheit than the method you used in Question 3?

5. Convert the given formula to determine degrees Fahrenheit. Show and explain your work.

6. The hottest temperature ever recorded on Earth occurred in Africa in 1922. It was recorded as 57.8°C. Use your formula to determine this temperature in degrees Fahrenheit.

7. Dry ice melts at −78°C. At what temperature in degrees Fahrenheit does dry ice melt?

8. Carlos and Mikala do not like working with fractions. They rewrite their equation so that it does not have fractions. Their work is shown.

Carlos

$$F = \frac{9}{5}C + 32$$
$$(5)F = 5\left(\frac{9}{5}C + 32\right)$$
$$5F = 9C + 160$$
$$5F - 9C = 160$$

Mikala

$$C = \frac{5}{9}(F - 32)$$
$$(9)C = (9)\frac{5}{9}(F - 32)$$
$$9C = 5(F - 32)$$
$$9C = 5F - 160$$
$$9C - 5F = -160$$

Carlos and Mikala got two different equations. Who is correct?

9. In the original equations, the coefficients $\frac{9}{5}$ and $\frac{5}{9}$ as well as the constant 32 had meaning based on temperature. What do the coefficients, 9 and 5 and the constant 160 represent in Carlos's and Mikala's equations?

Convert each given equation to the form indicated.
Then, identify the *x*-intercept, *y*-intercept, and the slope.
Show your work for each.

Remember, to convert to slope-intercept form, solve for *y*. To convert to standard form, get both variables on the same side and the constant on the other.

1. $6x + 5y = 20$

    a. slope-intercept form: **b.** *x*-intercept:

    c. *y*-intercept: **d.** slope:

2. $y = -\dfrac{2}{3}x + 10$

    a. standard form: **b.** *x*-intercept:

    c. *y*-intercept: **d.** slope:

3. $Ax + By = C$

    a. slope-intercept form: **b.** x-intercept:

    c. y-intercept: **d.** slope:

4. If you want to determine the y-intercept of an equation, which form is more efficient? Explain your reasoning.

5. If you want to determine the x-intercept of an equation, which form is more efficient? Explain your reasoning.

6. If you wanted to graph an equation on your calculator, which form is more efficient? Explain your reasoning.

Convert each literal equation to solve for the given variable.

1. Think Inside the Box is manufacturing new boxes for You Pack 'Em, We Ship 'Em (YPEWSE). YPEWSE told Think Inside the Box that the boxes must have a specific volume and area. However, YPEWSE did not specify a height for the boxes.

   a. Write a literal equation to calculate the volume of a box.

   b. Convert the volume formula to solve for height.

   c. YPEWSE specified the volume of the box must be 450 in^3 and the area of the base must be 75 in^2. Use your formula to determine the height of the new boxes.

2. The volume of an ice cream cone is the measure of how much ice cream a cone can hold. An ice cream cone company wants to make an ice cream cone with a larger radius that still holds the same amount of ice cream.

   a. Write an equation to calculate the volume of a cone.

   b. Convert the equation to solve for the radius.

3

3. Future value is the value of a sum of money at a specific date due to interest. The formula $A = P(1 + rt)$ is used to determine future value. The variable A is the future value, P is the principal, r is the interest rate, and t is the time.

A bank wants to know the interest rate of a customer's account who earned a certain amount of future value.

a. Convert the equation to solve for rate.

b. Jillian deposited $5000 in an account 10 years ago after her college graduation. The money she deposited now has a value of $15,000. Determine the interest rate of Jillian's account.

Be prepared to share your solutions and methods.

A Growing Business
Combining Linear Equations

LEARNING GOALS

In this lesson, you will:

- Write linear functions using the Distributive Property.
- Write and analyze a linear function as a combination of multiple linear functions.
- Interpret and understand component parts of functions.
- Analyze problem situations modeled by a combination of multiple linear functions.

You probably don't think about flowers on a daily basis, but there are some people who do! Florists routinely think about different types of flowers, arrangements of those flowers, ordering flowers, plants, balloons, baskets, and vases, and—phew! There's a lot to floristry! But make no mistake, the business of floristry is more than just flowers—it's dollars and cents and mathematics. For example, there are certain days of the years when there is a huge demand for roses, vases, and baby's breath. When this occurs, florists must accurately order roses and baby's breath in comparison to other flowers to make sure they can fulfill the demand, but not have a lot of these flowers left over.

What certain days do you think might have a higher demand for roses or vases? How do you think mathematics can help florists order and arrange flowers?

PROBLEM 1 Flowers for Sale!

Alexis is a flower vendor who grows and sells her own fresh-cut flowers. She has a stand in the city, and she distributes flowers to pedestrians during the day. She charges $5 for each flower, and each month she randomly gives away two flowers for free.

1. Identify the independent and dependent quantities in this problem situation.

2. Write a linear function, $a(x)$, to represent how much money Alexis earns each month. Use x to represent the number of flowers she sells each month. Write the function in simplest form.

> Remember, she gives away 2 flowers each month. How would you represent this information?

   a. $a(x) = $ _____

       $a(x) = $ _____

   b. What property did you use to write the simplified form of the function?

3. Describe the function. Is it increasing or decreasing? Is it discrete or continuous? Explain your reasoning.

4. Complete the table shown. First, determine the unit of measure for each expression. Then, describe the contextual meaning of each part of the function. Finally, choose a term from the word box to describe the mathematical meaning of each part.

> input value   output value   rate of change   *y*-intercept

Expression	Unit	What It Means	
		Contextual Meaning	Mathematical Meaning
a(*x*)			
5			
x			
(*x* − 2)			
−10			
5*x* − 10			

5. How much will Alexis earn in a month if she distributes 45 flowers? Show your work.

> In this situation, the word "distribute" refers to the flowers she sold and gave away.

6. How many flowers does Alexis distribute in a month if she earns $300? Show your work.

PROBLEM 2 Joining Forces

3

Bashir is also a flower vendor in a different part of the same city. He sells flowers for $3 each and gives away 4 flowers for free each month. He also earns an extra $6 each month by selling one of his homemade bracelets.

1. Write a linear function, $b(x)$, to represent the amount of money Bashir earns each month. Be sure to simplify your function.

$b(x) = $ _____

2. Consider the parts of Bashir's function and the parts of Alexis' function.

a. Explain the meaning of the rate of change and the y-intercept of each function.

b. Compare the units of the:

- output values,

- input values,

- rate of change, and

- *y*-intercepts

of *both* functions. What do you notice?

Bashir and Alexis decide to become business partners and combine their monthly earnings. They will each continue to sell to their own customers in different parts of the city. Bashir distributes twice as many flowers each month as Alexis.

3. At the end of the month when Alexis and Bashir combine their earnings, about how much will they will earn from each flower sold? Explain your prediction.

3

4. Suppose in one month Alexis distributes 20 flowers.

a. Use Alexis' function to calculate her earnings. Show your work.

b. Use Bashir's function to calculate his earnings. Show your work.

c. How much money would Bashir and Alexis make together if they combined their earnings?

d. Use your answer to part (c) to determine the average selling price of each flower after Alexis and Bashir combined their earnings. Does this match your prediction?

5. Nick tried to write a new function, $c(x)$, to represent Alexis' and Bashir's combined earnings. He said, "I can add the two functions like this:"

$$c(x) = a(x) + b(x)$$
$$c(x) = 5x - 10 + 3x - 6$$
$$= 8x - 16$$

Madison disagreed. She said, "That's not right. You can't add the functions because the x-values in the two functions don't mean the same thing, so they might be different values."

Who's correct—Madison or Nick? Explain your reasoning.

6. Use your answers to Question 4 and Nick's function to show why his function is not correct. Explain your method.

7. What does the slope represent in Nick's function? How does this compare with your answer to Question 4, part (d)?

Nick *could* actually add the two functions together. However, he did not recognize that the input values were different for Alexis and Bashir. To add two functions together, you must ensure the input values represent the same thing in both functions. A model can be used to represent the input values.

Let x represent the total number of flowers Alexis and Bashir distribute each month. The model shows that Bashir distributes twice as many flowers as Alexis each month and that together the number of flowers adds up to x.

$$x = \boxed{\text{Total Number of Flowers Distributed}}$$

$$= \frac{3}{3} = 1 \text{ whole}$$
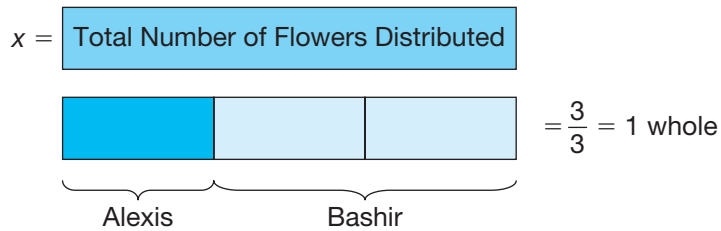
Alexis   Bashir

8. Write an expression to represent Alexis' share of the total flowers distributed. Then write an expression to represent Bashir's share of the total flowers distributed. Let x represent the total number of flowers distributed.

   Alexis:

   Bashir:

9. Rewrite Alexis' and Bashir's functions so they show each person's share of the total earnings. Then, add the functions to determine a new function, $c(x)$, that describes the combined amount of money Alexis and Bashir earn each month. Show your work.

10. What does the slope of the new function mean? What does the y-intercept of the new function mean?

The Silent Partner

Alexis and Bashir decide to partner with an investor. The investor will supply money for equipment, flower seeds, and other materials. In return, the investor will receive $0.50 for every flower distributed.

1. Write a new function, $d(x)$, to represent the amount of money the investor will earn each month from the total number of flowers distributed. Define what the variable x means in the function.

2. Write a function, $t(x)$, to represent the combined amount of money Alexis and Bashir will earn each month after they partner with the investor. Show your work.

3. In this case, why was it possible to determine a new function without rewriting $d(x)$?

4. Think about each problem situation and compare the functions $t(x)$ and $c(x)$. What do you notice?

Be prepared to share your solutions and methods.

Chapter **3** Summary

3.1 Using Linear Regression to Model Data and Make Predictions

Linear regression is the method of determining a line of best fit. A line of best fit is a line that is used to approximate data as close to as many points as possible. The equation that describes a line of best fit is called a linear regression equation. A graphing calculator can be used to determine the linear regression equation. When writing a linear regression equation based on the data, it is appropriate to use numbers that have the same number of significant decimal digits.

Example

A math teacher calculates the class average of each of his Algebra I classes between 2005 and 2012. Based on the data, he predicts that the average math score in his class in 2013 will be 85.19%.

Year	Average Math Score (%)	Data Point
2005	78.91	(1, 78.91)
2006	77.8	(2, 77.8)
2007	79.44	(3, 79.44)
2008	79.67	(4, 79.67)
2009	80.55	(5, 80.55)
2010	81.3	(6, 81.3)
2011	83.21	(7, 83.21)
2012	85.78	(8, 85.78)

$f(x) = 0.97x + 76.46$
$f(9) = 0.97(9) + 76.46$
$\quad = 85.19$

## 3.1 Interpreting the Correlation Coefficient of a Linear Regression Equation

The correlation coefficient shows how closely the data points are to forming a straight line and is represented by the variable r. It is often known as the r-value. The correlation coefficient is a value between -1 and 1. If the data show a positive correlation, the value of r is between 0 and 1. The closer the data are to forming a straight line, the closer the value of r is to 1. If the data show a negative correlation, the value of r is between 0 and -1. The closer the data are to forming a straight line, the closer the value of r is to -1. If there is no linear to the data, the value of r is 0.

Example

A math teacher calculates the class average of each of his Algebra I classes between 2005 and 2012. The graph of this data has a positive correlation. The correlation coefficient of $r = 0.922$ shows that the line of best fit is a good representation of the data.

Year	Average Math Score (%)	Data Point
2005	78.91	(1, 78.91)
2006	77.8	(2, 77.8)
2007	79.44	(3, 79.44)
2008	79.67	(4, 79.67)
2009	80.55	(5, 80.55)
2010	81.3	(6, 81.3)
2011	83.21	(7, 83.21)
2012	85.78	(8, 85.78)
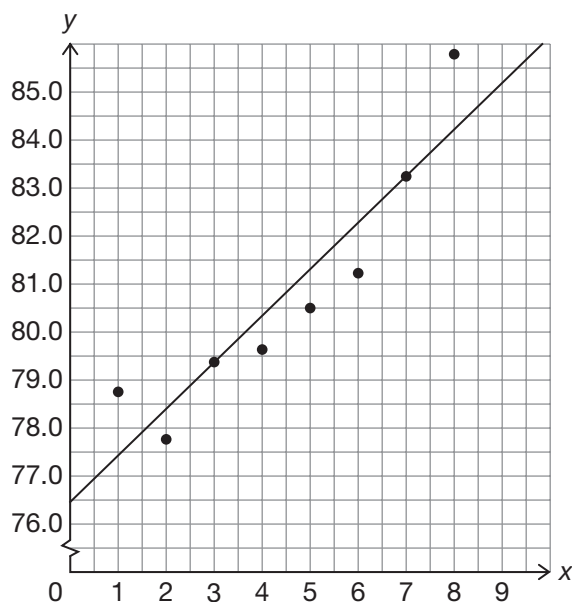
3.2 ## Writing and Solving a Function in Two Variables

When there are two different independent quantities in a problem situation, use two variables to represent them in an expression: $Ax + By$. The expression can be written in function notation as a function in two variables: $f(x, y) = Ax + By$.

Example

Karin makes and sells wreaths. She sells two sizes, large and small. Large wreaths cost $36 and small wreaths cost $24. If she sells 3 small wreaths and 2 large wreaths, she will make $144.

$f(g, s) = 36g + 24s$
$f(2, 3) = 36(2) + 24(3)$
$f(2, 3) = 144$

3.2 ## Identifying the *x*-intercept and *y*-Intercept of an Equation with Two Variables

3

When an equation with two variables equals a fixed amount, determine the *x*- and *y*-intercepts by replacing each variable with 0 and solving for the remaining variable.

Example

Karin makes and sells wreaths. She sells two sizes, large and small. Large wreaths cost $36 and small wreaths cost $24. She needs to make $288 to pay for supplies this week.

$$36g + 24s = 288 \qquad\qquad 36g + 24s = 288$$
$$36(0) + 24s = 288 \qquad\qquad 36g + 24(0) = 288$$
$$\frac{24s}{24} = \frac{288}{24} \qquad\qquad\qquad \frac{36g}{36} = \frac{288}{36}$$
$$s = 12 \qquad\qquad\qquad\qquad g = 8$$

If Karin sells no large wreaths, she must sell 12 small wreaths to make $288 for supplies.
If Karin sells no small wreaths, she must sell 8 large wreaths to make $288 for supplies.

Rewriting an Equation with Two Variables to Solve for One Variable

One way to solve for one variable is to rewrite the equation to isolate one variable on one side of the equation. These equations can then be used to determine the *x*-intercept and *y*-intercept which can be used to graph the equation.

3

Example

Solve for *g*.

$$36g + 24s = 288$$
$$36g = -24s + 288$$
$$g = -\frac{2}{3}s + 8$$
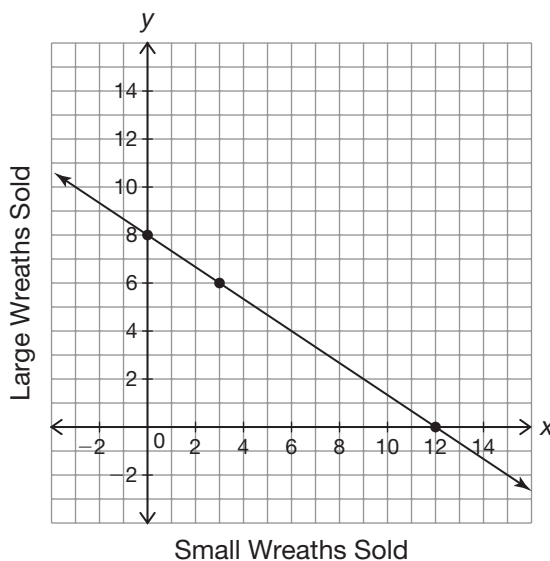
$$g = -\frac{2}{3}(0) + 8$$
$$g = 8$$
$$0 = -\frac{2}{3}s + 8$$
$$s = 12$$

The *x*-intercept is 12 and the *y*-intercept is 8.

Solve for *s*.
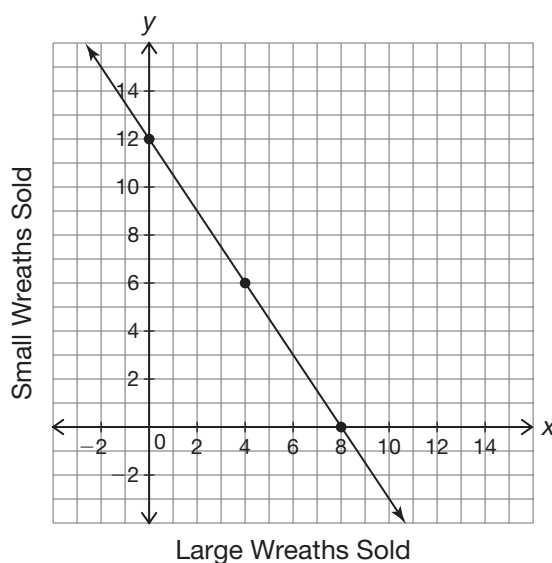
$$36g + 24s = 288$$
$$24s = -36g + 288$$
$$s = -\frac{3}{2}g + 12$$

$$s = -\frac{3}{2}(0) + 12$$
$$s = 12$$
$$0 = -\frac{3}{2}g + 12$$
$$g = 8$$

The *x*-intercept is 8 and the *y*-intercept is 12.

Small Wreaths Sold

Large Wreaths Sold

The slope is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 8}{3 - 0} = -\frac{2}{3}$$

The slope is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 12}{4 - 0} = -\frac{3}{2}$$

3.3 **Converting Literal Equations to Solve for a Specific Variable**

Literal equations are equations in which the variables represent specific measures. These equations can be manipulated to solve for one specific variable.

Example

The formula $A = r^2$ can be converted to solve for the radius r.

$$A = \pi r^2$$
$$\frac{A}{\pi} = r^2$$
$$r = \sqrt{\frac{A}{\pi}}$$

3.3 **Converting Equations Between Standard Form and Slope-Intercept Form**

Linear functions can be written in two forms, standard form and slope-intercept form. The standard form of a linear equation is $Ax + By = C$ where A, B, and C are constants and A and B are not both zero. All functions can be written in standard form. The slope-intercept form of a linear equation is $y = mx + b$ where b is the y-intercept and m is the slope. It is often necessary to change between the forms as one form might be more appropriate to represent a situation. Remember, to convert to slope-intercept form, solve for y. To convert to standard form get both variables on the same side and the constant on the other.

Example

$$3x + 7y = 28$$

Slope-Intercept Form :

$$3x + 7y = 28$$
$$7y = -3x + 28$$
$$\frac{7y}{7} = \frac{-3x}{7} + \frac{28}{7}$$
$$y = -\frac{3}{7}x + 4$$

x-intercept:

$$3x + 7y = 28$$
$$3x + 7(0) = 28$$
$$\frac{3x}{3} = \frac{28}{3}$$
$$x = \frac{28}{3}$$

y-intercept:

$$y = -\frac{3}{7}x + 4$$
$$y = -\frac{3}{7}(0) + 4$$
$$y = 4$$

Slope:

$$y = -\frac{3}{7}x + 4$$
$$m = -\frac{3}{7}$$

Writing a Linear Equation Using the Distributive Property

Use the Distributive Property of Multiplication to write a linear equation in two different ways. Use the best form to solve for different situations.

Example

As a reward for being named Employee of the Month, Tito's boss will double his income for the last week of the month. Tito earns a weekly flat rate of $100 plus any money he makes on commissions. The function representing this situation is $f(x) = 2(100 + x)$ or $f(x) = 200 + 2x$.

Tito earns $400 this week. He earns $100 on commission.

$$f(x) = 2(100 + x)$$
$$400 = 2(100 + x)$$
$$\frac{400}{2} = \frac{2(100 + x)}{2}$$
$$200 = 100 + x$$
$$100 = x$$

Tito earns $50 on commission. He earns $300 total this week.

$$f(x) = 200 + 2x$$
$$f(x) = 200 + 2(50)$$
$$f(x) = 300$$

Combining Linear Equations Using the Distributive Property

When combining two functions, consider the meaning of each function's *x*-value. Combine the functions by determining each original *x*-value's share of the new *x*-value. Rewrite the original equations in terms of this shared *x*-value and then combine.

Example

Maya makes and sells cupcakes at a small shop in her neighborhood. She eats 2 per day randomly for quality control. The cupcakes cost $4 each.

$m(x) = 4(x - 2) = 4x - 8$

Ping opens a franchise in a different neighborhood and decides to charge $5 for each cupcake. He also makes an extra $20 a week from a local artist whom he lets display her pieces in his shop.

$p(x) = 5x + 20$

Let *x* represents the number of cupcakes each baker makes. Maya has a more efficient kitchen and makes twice as many cupcakes as Ping.

Maya contributes two-thirds and Ping contributes one-third of their combined cupcakes. The slope means that combined, the bakers can make about $4.33 per cupcake. The *y*-intercept means that despite Maya eating cupcakes for quality control, they will earn $12 a week from the local artist.

$$m(x) = 4\left(\frac{2}{3}x\right) - 8$$

$$p(x) = 5\left(\frac{1}{3}x\right) + 20$$

$$c(x) = m(x) + p(x)$$

$$c(x) = 4\left(\frac{2}{3}x\right) - 8 + 5\left(\frac{1}{3}x\right) + 20$$

$$= \frac{8}{3}x - 8 + \frac{5}{3}x + 20$$

$$= \frac{13}{3}x + 12$$