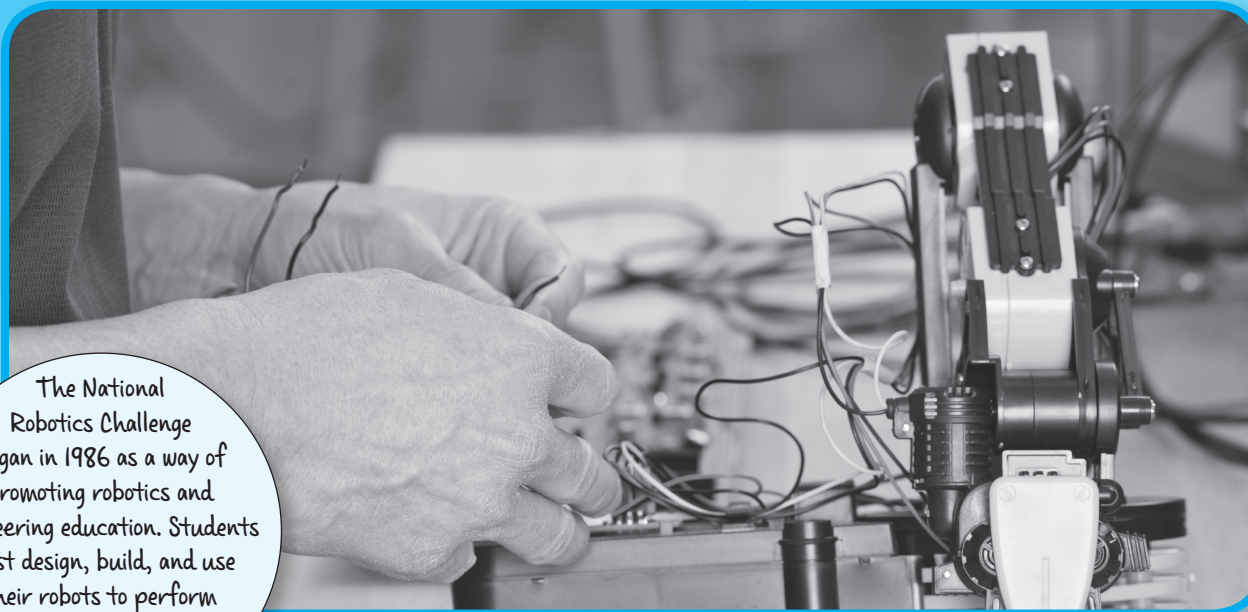# Systems of Equations

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The National Robotics Challenge began in 1986 as a way of promoting robotics and engineering education. Students must design, build, and use their robots to perform different tasks, including obstacles courses!

Prepping for the Robot Challenge

Solving Linear Systems Graphically and Algebraically

LEARNING GOALS

In this lesson, you will:

- Write systems of linear equations.
- Graph systems of linear equations.
- Determine the intersection point, or break-even point, from a graph.
- Use the substitution method to determine the intersection point.
- Understand that systems of equations can have one, zero, or infinite solutions.

KEY TERMS

- break-even point
- system of linear equations
- substitution method
- consistent systems
- inconsistent systems

In today's world, the field of robotics is growing rapidly; however the interest and fascination with robots has been occurring for centuries. The word *robot* was first used in a 1920 play describing a factory that creates artificial people which could be mistaken for humans. However, the idea of *robots* did not begin there.

Descriptions of some of the first automatons, or self-operating machines, were recorded as early as the 3rd century BC! An ancient Chinese text describes a mechanical engineer presenting King Mu of Zhou with a life-size, human-shaped figure that could walk, move its head, and sing. By the 1200s, a Muslim engineer named al-Jazari created some of the first human-like machines that could be used for practical purposes. He created a drink-serving waitress and a hand-washing automaton that were both functional using hydropower. It is amazing to think of these people inventing such incredible robots without the use of today's technology.

Are you surprised to learn that people created robots so long ago? What do we use robots for today? What do you think robots will be able to do in the future?

Gwen has a part-time job working at Reliable Robots (RR) which sells electronics and hardware parts for robot creators. One of her tasks is to analyze RR's finances in terms of cost and income. Her boss, Mr. Robo, asks her to determine the *break-even point* for the cost and the income. The **break-even point** is the point when the cost and the income are equal. Gwen begins with the income and costs for gearboxes.

1. Let the function $I(g)$ represent the income (I) from selling gearboxes (g) and the function $C(g)$ represent the cost (C) of purchasing gearboxes (g).

   a. Describe the relationship between the income function and the cost function that will show the break-even point. Explain your reasoning.

   b. Describe the relationship between the income function and the cost function that will show a profit from selling gearboxes. Explain your reasoning.

2. RR purchases gearboxes from The Metalists for $5.77 per gearbox plus a one-time credit check fee of $45.00. RR sells each gearbox for $8.50.

   a. Write the function for the income generated from selling gearboxes.

   b. Write the function for the cost of purchasing gearboxes from The Metalists.

3. Sketch a graph of each function on the coordinate plane to predict the break-even point of the income from RR selling the gearboxes and the cost of purchasing the gearboxes.
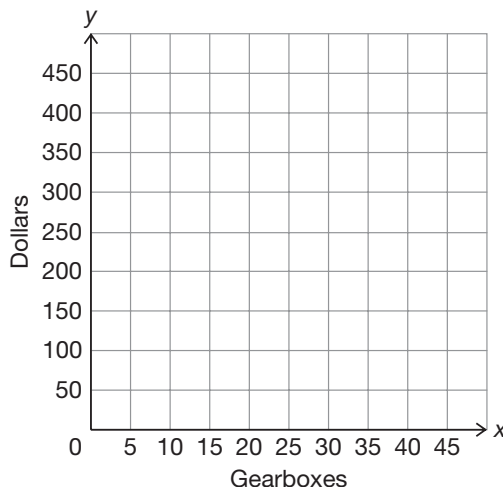
Be sure to label each graph so you know which graph represents cost and which represents income.

a. How is the break-even point for $I(g)$ and $C(g)$ represented on the graph you sketched? Estimate the break-even point.

b. Could you determine the exact break-even point from the graph? Why or why not.

As you learned previously, the coordinates of an intersection point of two graphs can be exact or approximate depending on whether the intersection point is located on the intersection of two grid lines. You also learned that you had to use algebra to prove an exact intersection point.

Notice the units of measure for the independent and dependent quantities in $I(g)$ and $C(g)$ are the same.

In both functions $I(g)$ and $C(g)$, the g represents the independent quantity, gearboxes, and the dependent quantity is dollars. However, the dollars in each function represent different "types" of dollars. You know that dollars are represented differently because you determined two different functions—one function for cost (in dollars) and one function for income (in dollars).

> Recall that in A *New Way to Write Something Familiar* in Chapter 1 Lesson 3, you transformed a function in equation form into function notation to more efficiently represent the independent and dependent quantities.

When determining the break-even point algebraically between two functions, it is more efficient to transform each function into equation form. In this case, by transforming the functions into equation form, you establish one unit of measure for the dependent quantity: dollars.

Analyze the functions representing cost and income from gearboxes for Reliable Robots.

$$I(g) = 8.5g \qquad C(g) = 5.77g + 45$$

Since g is the independent variable, you can represent g as x in equation form.

$$I(x) = 8.5x \qquad C(x) = 5.77x + 45$$

Since both $I(g)$ and $C(g)$ represent the dependent quantity in dollars, you can represent each using y as the variable.

$$y = 8.5x \qquad y = 5.77x + 45$$

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4. Do you think it is possible to use other variables instead of x and y when transforming a function written in function notation to equation form?

When two or more equations define a relationship between quantities, they form a **system of linear equations**.

5. What is the relationship between the two equations in this problem situation?

> Remember, a solution for a system of linear equations occurs when the values of the variables satisfy *all* of the linear equations.

Now that you have successfully created a system of linear equations, you can determine the break-even point for the gearboxes at RR. One way to solve a system of linear equations is called the *substitution method*. The **substitution method** is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

Consider the system of equations from the previous worked example.

$$\begin{cases} y = 5.77x + 45 \\ y = 8.5x \end{cases}$$

> Since both equations are equal to y, we can set the equations equal to each other.

Substitute the variable y in the first equation with the equivalent expression in the second equation.

$$8.5x = 5.77x + 45$$

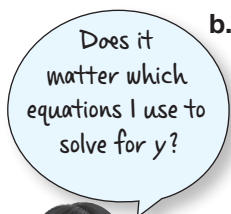Isolate the variable to solve.

$$2.73x = 45$$

$$x \approx 16.48$$

6. Analyze the solution $x \approx 16.48$.

    a. What does this point represent in terms of the problem situation? Why is this solution an approximation?

    b. Solve for y. Describe the solution in terms of this problem situation.

> Does it matter which equations I use to solve for y?

    c. What is the profit from gearboxes at the break-even point?

    d. Does this break-even point make sense in terms of the problem situation? Why or why not.

7. Analyze your graph of the cost and the income for the different number of gearboxes.

    a. Draw a box around the portion of the graph that represents when RR is losing money. Then write an inequality to represent this portion of the graph and describe what it means in terms of the problem situation.

b. Draw an oval around the portion of the graph that represents when RR is earning money. Then write an inequality to represent this portion of the graph and describe what it means in terms of the problem situation.

c. Write an equation to represent the portion of the graph that represents when RR breaks even and describe what it means in terms of the problem situation.

PROBLEM 2 Saving Up

Marcus and Phillip are in the Robotics Club. They are both saving money to buy materials to build a new robot. They plan to save the same amount of money each week.

1. Write a function to represent the time it takes Marcus and Phillip to save money. Define your variables and explain why you chose those variables.

Marcus decides to open a new bank account. He deposits $25 that he won in a robotics competition. He also plans on depositing $10 a week that he earns from tutoring. Phillip decides he wants to keep his money in a sock drawer. He already has $40 saved from mowing lawns over the summer. He plans to also save $10 a week from his allowance.

2. Write a function to represent the information regarding Marcus and Phillip saving money for new robotics materials.

3. Predict when Marcus and Phillip will have the same amount of money saved. Use your functions to help you determine your prediction.

You can prove your prediction by solving and graphing a system of linear equations.

4. Rewrite each function as an equation. Use *x* and *y* for the variables of each function in equation form and define the variables. Then, write a system of linear equations.

5. Analyze each equation.

   **a.** Describe what the slope of each line represents in this problem situation.

    b. How do the slopes compare? Describe what this means in terms of this problem situation.

    c. Describe what the *y*-intercept of each line represents in this problem situation.

    d. How do the *y*-intercepts compare? Describe what this means in terms of this problem situation.
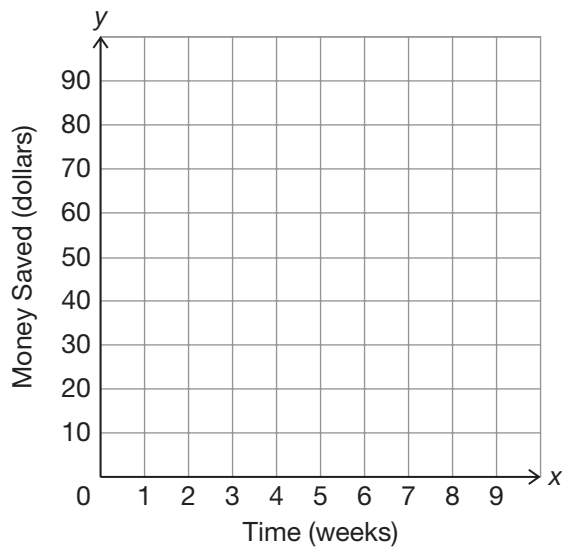
6. Determine the solution of the system of linear equations algebraically and graphically.

    a. Use the substitution method to determine the intersection point.

> Can you have a solution without variables?

    b. Does your solution make sense? Describe what this means in terms of the problem situation.
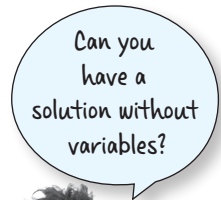
c. Predict what the graph of this system will look like. Explain your reasoning.

d. Graph both equations on the coordinate plane provided.

(graph: y-axis labeled "Money Saved (dollars)" marked 10 to 90; x-axis labeled "Time (weeks)" marked 0 to 9)

7. Analyze the graph you created.

    a. Describe the relationship between the graphs.

    b. Does this linear system have a solution? Explain your reasoning.

8. Was your prediction in Question 3 correct? Explain how you algebraically and graphically proved your prediction.
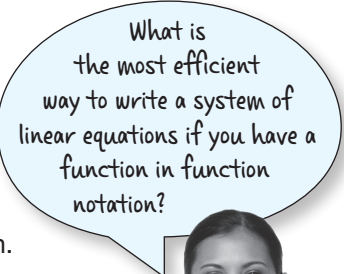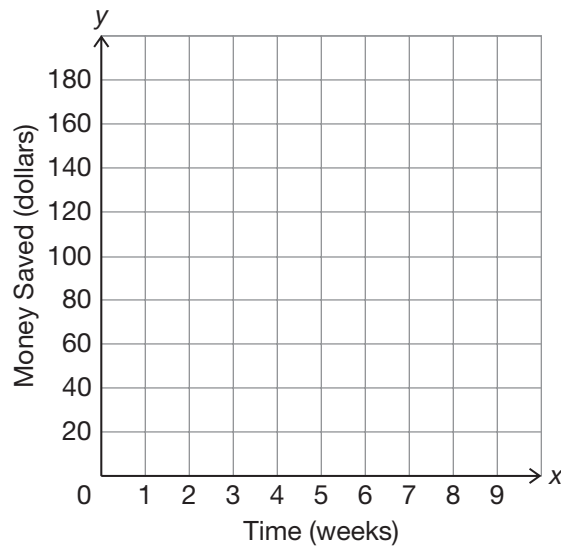
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9. Tonya is also in the Robotics Club and has heard about Marcus's and Phillip's savings plans. She wants to be able to buy her new materials before Phillip, so she opens her own bank account. She is able to deposit $40 in her account that she has saved from her job as a waitress. Each week she also deposits $4 from her tips.

a. Write a function that represents the information about Tonya saving money every week. Do not forget to define your variables.

b. Write a linear system to represent the total amount of money Tonya and Phillip have after a certain amount of time.

> What is the most efficient way to write a system of linear equations if you have a function in function notation?

c. Graph the linear system on the coordinate plane shown.

10. Do the graphs intersect? If so, describe the meaning in terms of this problem situation.

11. Phillip and Tonya went on a shopping spree this weekend and spent all their savings except for $40 each. Phillip is still saving $10 a week from his allowance. Tonya now deposits her tips twice a week. On Tuesdays she deposits $4 and on Saturdays she deposits $6. Phillip claims he is still saving more each week than Tonya.

a. Do you think Phillip's claim is true? Explain your reasoning.

6

b. How can you prove your prediction?

**12.** Prove your prediction algebraically and graphically.

a. Write functions that represent any new information about the way Tonya and Phillip are now saving money.

When naming your functions, be sure you can tell the difference between these functions and the ones you previously wrote. If you used $P(w)$ to represent the amount of money that Phillip is saving each week, you might consider using subscripts like $P_2(w)$ to represent the new function for the money he is saving each week.

b. Write a new linear system to represent the total amount of money each friend has after a certain amount of time.

c. Graph the linear system on the coordinate plane.
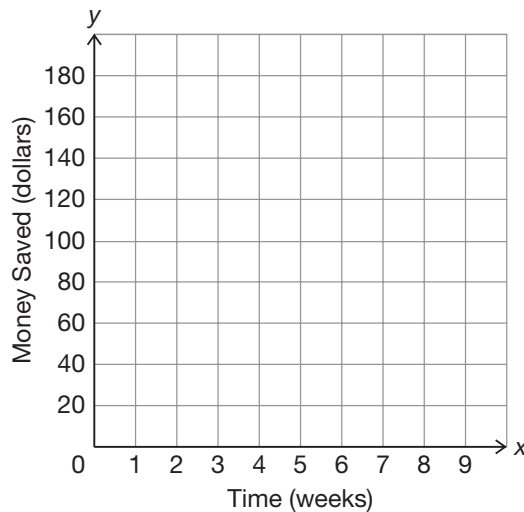
13. Analyze the graph.

a. Describe the relationship between the graphs. What does this mean in terms of this problem situation?

b. Algebraically prove the relationship you stated in part (a).

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c. Does this solution prove the relationship? Explain your reasoning.

14. Was Phillip's claim that he is still saving more than Tonya a true statement? Explain why or why not.

PROBLEM 3 Transforming Equations: More Than Meets the Eye

Not all systems will be written in slope-intercept form or function notation. Systems can also be written in standard form. Let's explore a system in standard form.

$$\begin{cases} 2x + 8y = 10 \\ 4x = y - 2 \end{cases}$$

Do you think there is more than one way to transform one of the equations in the system to create a new equation with only one unknown?

1. Analyze each student's work.

Dontrell

$$\begin{cases} 2x + 8y = 10 \\ 4x = y - 2 \end{cases}$$

$2x + 8y = 10$
$4x + 2 = y$

$2x + 8(4x + 2) = 10$

Janelle

$$\begin{cases} 2x + 8y = 10 \\ 4x = y - 2 \end{cases}$$

$2x = 10 - 8y$
$x = 5 - 4y$

$4(5 - 4y) = y - 2$

Maria

$$\begin{cases} 2x + 8y = 10 \\ 4x = y - 2 \end{cases}$$

$8y = -2x + 10$ $4x = y - 2$

$y = -\frac{2}{8}x + \frac{10}{8}$ $4x + 2 = y$

$-\frac{2}{8}x + \frac{10}{8} = 4x + 2$

a. Describe the method Dontrell used to solve this system of equations and explain why he is correct.

6

b. Describe the method Janelle used to solve this system of equations and explain why her reasoning is correct.

c. Describe the method Maria used to solve this system of equations and explain why her reasoning is correct.

2. Which method do you prefer for solving this system of equations?

3. Use one of the methods shown or use your own method to determine the solution to this system of equations.

6

4. Soo Jin encountered this system of linear equations.

$$\begin{cases} 3.5x + 1.2y = 8 \\ 4.7x + 0.3y = 10.3 \end{cases}$$

However, Soo Jin has decimaphobia—a fear of decimals! Sammy tells her she has nothing to fear. He says, "All you need to do is multiply each equation by 10 to transform the system into whole numbers."

a. Is Sammy correct? Explain why or why not.

> Soo Jin! There is no such thing as decimaphobia!

b. Soo Jin attempts Sammy's method. Her work is shown.

> **Soo Jin**
>
> $$\begin{cases} 35x + 12y = 8 \\ 47x + 3y = 103 \end{cases}$$

Explain the mistake(s) Soo Jin made and then determine the correct way to rewrite this system.

6

Talk the Talk

1. Use any method of substitution to determine the solutions for each of the systems of linear equations.

a. $\begin{cases} 8x - 2y = 7 \\ 2x + y = 4 \end{cases}$

b. $\begin{cases} 0.4x + 0.3y = 1 \\ 0.1y = 0.2x \end{cases}$

c. $\begin{cases} \dfrac{1}{2}x + \dfrac{1}{4}y = 6 \\ y = 4 \end{cases}$

d. $\begin{cases} 6x + 3y = 5 \\ y = -2x + 1 \end{cases}$

6

A system of equations may have one unique solution, infinitely many solutions, or no solution. Systems that have one or many solutions are called **consistent systems**. Systems with no solution are called **inconsistent systems**.

2. Complete the table.

	Consistent Systems		Inconsistent Systems
Number of Solutions			
Description of *y*-intercepts	*y*-intercepts can be the same or different	*y*-intercepts are the same	*y*-intercepts are different
Description of Graph			

6

Be prepared to share your solutions and methods.

6

There's Another Way?
Using Linear Combinations to Solve a Linear System

LEARNING GOALS

In this lesson, you will:

- Write a system of equations to represent a problem context.
- Solve a system of equations algebraically using linear combinations (elimination).

KEY TERM

- linear combinations method

Morse code is a communication system which allows people to "speak" with sound. Words are transmitted using short sounds called "dits," which are represented in writing as dots; while long sounds, called "dahs," are represented in writing as dashes. The letters of the alphabet and digits each have their own unique collection of dits and dahs:

When you combine these codes, you can produce sentences in Morse code.

Try it out! Communicate with your friends using Morse code.

PROBLEM 1   People Love Their Comics—Even On-Line!

There are a total of 324 people who joined the Comic Gurus group on a social media site. Female group members outnumber males by 34. Determine how many males and females joined the Comic Gurus group.

1. Write an equation in standard form that represents the total number of people who joined the Comic Gurus group. Use x to represent the female members, and use y to represent the male members.

2. Write an equation in standard form to represent the number of female members in relationship to the number of male members.

3. How are these equations the same? How are the equations different?

4. Complete parts (a) through (e) to write and solve a linear system of equations for this problem situation.

   a. Write a linear system for this problem situation.

   b. Add the two equations together.

   c. Solve the resulting equation.

> I see how you can add equations. $(4 + 2) = 6$ $(4 - 2) = 2$ So, if I can add 6 and 2 and get 8, then that means I can add $(4 + 2)$ and $(4 - 2)$ and get 8 also. So, $(4 + 2) + (4 - 2) = 8$.

d. Substitute the *x*-value that you obtained in part (c) into one of the original equations and solve to determine the value of *y*.

e. What is the solution of the linear system? Check your solution algebraically.

5. Interpret the solution of the linear system in this problem situation.

6. What effect did adding the equations together have?

7. Describe how the coefficients of *y* in the original system are related.

6

PROBLEM 2 Let It Snow . . . For Winter Get-Aways

Let It Snow Resort offers two winter specials: the Get-Away Special and the Extended Stay Special. Let It Snow claims that the Extended Stay Special is the better deal. The Get-Away Special offers two nights of lodging and four meals for $270. The Extended Stay Special offers three nights of lodging and eight meals for $435. Determine if the Extended Stay Special is the better deal.

1. Write an equation in standard form that represents the Get-Away Special. Let n represent the cost for one night of lodging at the resort, and let m represent the cost for each meal.

2. Write an equation in standard form that represents the Extended Stay Special. Use the same variables you used in Question 1.

3. How are these equations the same? How are these equations different?

4. Complete parts (a) through (h) to write and solve the system comparing the two winter specials.

   a. Multiply each side of the equation that represents the Get-Away Special by -2. Simplify the equation; maintain standard form.

   > If I multiply both sides of an equation by the same number, is the equation still true?

   b. Write a linear system of equations using the transformed equation you wrote that represents the Get-Away Special and the equation that represents the Extended Stay Special.

   c. How do the coefficients of the equations in your linear system of equations compare?

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d. Add the equations in your linear system together. Then simplify the result. What does the result represent?

> When you divide a negative value by -1, you make it positive.

e. How will you determine the *m*-value of the linear system?

f. Determine the value of *m* for the linear system.

g. What is the solution of the linear system? Interpret the solution of the linear system in the problem situation.

h. Check your solution algebraically.

5. Is the Extended Stay Special the better deal? Explain why or why not.

6

The algebraic method you used to solve the linear systems in Problems 1 and 2 is called the *linear combinations method*. The **linear combinations method** is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable. You can then determine the value of that variable and use it to determine the value of the other variable.

In many cases, one or both of the equations in the system must be multiplied by a constant so that when the equations are added together, the result is an equation in one variable. This means that the coefficients of either the term containing x or y must be opposites.

Let's consider a system where neither of the x- or y-terms are opposites.

$$\begin{cases} 4x + 2y = 3 \\ 5x + 3y = 4 \end{cases}$$

Multiply each equation by a constant that results in opposite coefficients for one of the variables.

$$3(4x + 2y) = 3(3)$$
$$-2(5x + 3y) = -2(4)$$

$$12x + 6y = 9$$
$$-10x - 6y = -8$$

Now that the y-values are opposites, you can solve this linear system.

1. Solve the new linear system shown in the worked example.

> You can check your solution by substituting the ordered pair back into the original equations.

2. Describe the first step needed to solve each system using the linear combination method. Identify the variable that will be solved when you add the equations.

   a. $5x + 2y = 10$ and $3x + 2y = 6$

   > Are there other ways to create opposite coefficients for either variable?

   b. $x + 3y = 15$ and $5x + 2y = 7$

   c. $4x + 3y = 12$ and $3x + 2y = 4$

6

3. Solve each system using linear combinations.

a. $\begin{cases} 2x + y = 8 \\ 3x - y = 7 \end{cases}$

b. $\begin{cases} 4x + 3y = 24 \\ 3x + y = -2 \end{cases}$

c. $\begin{cases} 3x + 5y = 17 \\ 2x + 3y = 11 \end{cases}$

Be prepared to share your solutions and methods.

6

6

What's for Lunch?
Solving More Systems

In this lesson, you will:

- Write a linear system of equations to represent a problem context.
- Solve a linear system of equations using the linear combinations method.

In most large cities throughout the world, you can find people selling food out of mobile food carts. These food carts can have an advantage over permanent restaurants because sellers can move the cart from location to location to try and increase sales. While this mobility can be beneficial, some carts do become permanent restaurants. For example, in 1939, Paul and Betty Pink purchased a lunch cart and pushed it near the present day corner of Melrose and La Brea in Los Angeles. As business grew, so did Pink's and by 1946 it was located in a permanent building on North La Brea where it still stands. Now, Pink's Hot Dogs has opened other locations in California, Nevada, and even Ohio. Their success also has stemmed into being referenced or seen on-camera in dozens of television shows and films.

Which do you think would earn more money—a food cart or a permanent restaurant? Which would be cheaper to run?

Constance owns a small lunch cart. She changes her menu daily. Yesterday, she offered a chef salad for $5.75 or a hoagie for $5.00. She sold 85 lunches for a total of $464. Determine how many chef salads and hoagies she sold.

1. Write an equation in standard form that represents the total number of lunches in terms of the number of chef salads sold and the number of hoagies sold. Let x represent the number of chef salads sold, and let y represent the number of hoagies sold.

2. Write an equation in standard form that represents the amount of money collected. Use the same variables as those used in Question 1.

3. Write a system of linear equations to represent this problem situation.

4. What methods can you use to solve this system of linear equations?

> Think about all the strategies you have used in previous lessons.

5. Determine the solution of this linear system of equations by using linear combinations. Then, check your answer.

6. Interpret your solution to the linear system in terms of this problem situation.

The School Spirit Club is making beaded friendship bracelets with the school colors to sell in the school store. The bracelets are black and orange and come in two lengths: 5 inches and 7 inches. The club has enough beads to make a total of 84 bracelets. So far, they have made 49 bracelets, which represents $\frac{1}{2}$ the number of 5-inch bracelets plus $\frac{3}{4}$ the number of 7-inch bracelets they plan to make and sell. Determine how many 5-inch and 7-inch bracelets the club plans to make.

1. Write an equation in standard form that represents the total number of bracelets the School Spirit Club can make out of the beads that they have. Let x represent the number of 5-inch bracelets, and let y represent the number of 7-inch bracelets.

2. Write an equation in standard form that represents the number of bracelets the School Spirit Club has made so far. Use the same variables as those used in Question 1.

3. Write a system of linear equations that represents this problem situation.

4. Karyn says that the first step she would take to solve this system would be to first multiply the second equation by the least common denominator (LCD) of the fractions. Is she correct? Explain your reasoning.

5. Rewrite the equation containing fractions as an equivalent equation without fractions.

6. Determine the solution to the system of equations by using linear combinations and check your answer.

7. Interpret the solution of the linear system in terms of this problem situation.

Talk the Talk

1. Solve each linear system using linear combinations. Check all solutions.

a. $\begin{cases} x + 2y = 2 \\ 5x - 3y = -29 \end{cases}$

b. $\begin{cases} \dfrac{1}{2}x + \dfrac{1}{3}y = 3 \\ 3x + 5y = 36 \end{cases}$

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c. $\begin{cases} 0.6x + 0.2y = 2.2 \\ 0.5x - 0.2y = 1.1 \end{cases}$

d. $\begin{cases} \dfrac{1}{2}x + \dfrac{3}{5}y = 17 \\ \dfrac{1}{5}x + \dfrac{3}{4}y = 17 \end{cases}$

2. You have used three different methods for solving systems of equations: graphing, substitution, and linear combinations. Describe how to use each method and the characteristics of the system that makes this method most appropriate.

- Graphing Method:

- Substitution Method:

- Linear Combinations Method:

Which method do you like best?

Be prepared to share your solutions and methods.

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6

Which Is the Best Method?

Using Graphing, Substitution, and Linear Combinations

LEARNING GOALS

In this lesson you will:

- Use various methods of solving systems of linear equations to determine the better paying job.
- Use various methods of solving systems of linear equations to determine the better buy.

Successful businesses spend and earn large amounts of money. But how do they know what to spend their money on and how much money to spend? Often, these decisions are made by financial analysts. These professionals analyze financial records of the business and use them to help decide what financial decisions should be made. These financial records may include past performance of the company as well as comparisons between their company and other similar companies. The analysts may also try to predict future performance of the company using extrapolation. This analysis is then used to make decisions such as continuing or discontinuing a main part of the business, making or purchasing materials, or deciding whether to invest or lend any of their earnings. Of course, a financial analyst's goal is always to help the company make money, pay off any debts, and ensure the company remains stable for the long run.

What sort of pressures do you think financial analysts face day to day at their job? What mathematical knowledge might they need to know to make informed decisions? Is this a job you think you might be good at? Why or why not?

PROBLEM 1 Savin' on Cruisin'

The Bici Bicycle Company is planning to make a low price ultra-light bicycle. There are two different plans being considered for building this bicycle. The first plan includes a cost of $125,000 to design and build a prototype bicycle. The materials and labor costs for each bike made under the first plan will be $225. The second plan includes a cost of $100,000 to design and build the prototype. The materials and labor costs for each bike made under the second plan will be $275.

You recently got a job at Bici Bicycle Company as a financial analyst. You have been asked to analyze the costs for each proposed bicycle prototype and determine which plan Bici should follow. Use any method to determine your response.

PROBLEM **2**   We Offer Free Calls and Super Low Prices on Texts!

Demetrius is in search of a new cell phone plan. He is considering two different cell services from two different providers.

Bouncing Cell Service offers a monthly fee of $99.99 and 200 free text messages per month. Once a user exceeds the free monthly number of text messages, each subsequent text message is $0.05 per text. Rolling Cell Service offers a monthly fee of $79.99 and 150 free text messages per month. Once a user exceeds the free monthly number of text messages, each subsequent text message is $0.08 per text. Demetrius is unsure which plan to choose. He wasn't very careful with his last contract and paid a lot of extra money in charges for texts.

Write an email to advise Demetrius which plan to choose. Use any method to determine your response.

6

I've Got a Knack for Sales, But Which Company Should I Choose?

Jose interviewed for two different sales positions at competing companies. Reliable Robotics has offered Jose a salary of $31,200 per year, plus a 9% commission on the total sales he makes per week. Robot Renegades will offer him $26,000 per year, plus a 15% commission on the total sales per week.

Jose isn't sure which offer to accept. He's great at making a sale, but he's just not sure which job will be better in terms of his pay. He is confident that he can make $2000 worth of sales each week.

Write an email to Jose with your recommendation of which job offers better compensation. Use any method to determine your response.

Be prepared to share your solutions and methods.

Chapter **6** Summary

KEY TERMS

- break-even point (6.1)
- system of linear equations (6.1)

- substitution method (6.1)
- consistent systems (6.1)
- inconsistent systems (6.1)

- linear combinations method (6.2)

6.1 **Predicting the Solution of a System Using Graphing**

Functions can be written using the same units of measure, but the functions may represent different things. When this occurs, functions can be transformed into equation form to ensure the same variables are used. When two or more linear equations define a relationship between quantities, they form a system of linear equations. The solution of a linear system is an ordered pair (x, y) that is a solution to both equations in the system. One way to predict the solution to a system of equations is to graph both equations and identify the point where the two graphs intersect.

Example

The McCarter family is determining whether they want to buy a summer pass to the local swimming pool. Without a summer pass, they must pay $12.50 per visit to the pool. If they buy the $50 summer pass, then they must pay $7.50 per visit to the pool.
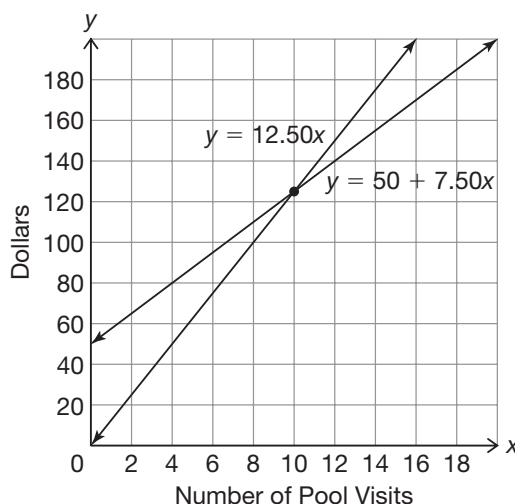
Let v represent visits to the pool.

$N(v) = 12.50v$

$P(v) = 50 + 7.50v$

A system of equations can now be used to determine when owning the summer pass is cheaper than paying per visit.

$$\begin{cases} y = 12.50x \\ y = 50 + 7.50x \end{cases}$$

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The intersection point appears to be (10, 125). If the McCarters visit the pool more than 10 times, they will save money with the summer pass. If they visit the pool fewer than 10 times, then they will save money by paying per visit.

6.1 Solving Systems of Linear Equations Using the Substitution Method

The substitution method is a process of solving equations by substituting a variable in one equation with an equivalent expression. The value of one variable is then determined by isolating the variable and solving the equation. This value is then used to determine the value of the other variable. It is generally more efficient to transform any functions in functional notation into equation form.

Example

$$f(p) = 4p - 10$$
$$y = 4x - 10$$
$$g(p) = -1.5p + 1$$
$$y = -1.5x + 1$$

$$\begin{cases} y = 4x - 10 \\ y = -1.5x + 1 \end{cases}$$

$$4x - 10 = -1.5x + 1$$
$$4x + 1.5x - 10 = -1.5x + 1.5x + 1$$
$$5.5x - 10 = 1$$
$$5.5x - 10 + 10 = 1 + 10$$
$$5.5x = 11$$
$$\frac{5.5x}{5.5} = \frac{11}{5.5}$$
$$x = 2$$

$$y = 4(2) - 10$$
$$y = 8 - 10$$
$$y = -2$$

The solution is (2, −2).

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Solving a System of Equations Using the Linear Combinations Method

The linear combinations method is a process used to solve a system of equations by adding two equations together so that they result in an equation with one variable. Once the value of this variable is determined, it can be used to determine the value of the other variable. In many cases, one or both of the equations may need to be multiplied by a constant so that the coefficients of the term containing either x or y are opposites. Then when the equations are added together, the result is an equation in one variable.

Examples

$5x + 2y = 16$ and $2x + 6y = 22$

$$-3(5x + 2y) = -3(16)$$
$$2x + 6y = 22$$

$$-15x - 6y = -48$$
$$\underline{2x + 6y = 22}$$
$$-13x = -26$$
$$\frac{-13x}{-13} = \frac{-26}{-13}$$
$$x = 2$$

$$5x + 2y = 16$$
$$5(2) + 2y = 16$$
$$10 + 2y = 16$$
$$2y = 6$$
$$y = 3$$

Check:

$$5(2) + 2(3) = 16$$
$$10 + 6 = 16$$
$$16 = 16 ✔$$

$$2(2) + 6(3) = 22$$
$$4 + 18 = 22$$
$$22 = 22 ✔$$

The solution is (2, 3).

$$\frac{1}{2}x + \frac{1}{3}y = 7$$
$$\frac{1}{4}x - \frac{1}{9}y = 1$$

$\longrightarrow$

$$12\left(\frac{1}{2}x + \frac{1}{3}y = 7\right)$$
$$36\left(\frac{1}{4}x - \frac{1}{9}y = 1\right)$$

$\longrightarrow$

$$6x + 4y = 84$$
$$9x - 4y = 36$$

$$6x + 4y = 84$$
$$\underline{9x - 4y = 36}$$
$$15x = 120$$
$$\frac{15x}{15} = \frac{120}{15}$$
$$x = 8$$

$$6x + 4y = 84$$
$$6(8) + 4y = 84$$
$$48 + 4y = 84$$
$$\frac{4y}{4} = \frac{36}{4}$$
$$y = 9$$

Check:

$$\frac{1}{2}(8) + \frac{1}{3}(9) = 7$$
$$4 + 3 = 7$$
$$7 = 7 ✔$$

$$\frac{1}{4}(8) - \frac{1}{9}(9) = 1$$
$$2 - 1 = 1$$
$$1 = 1 ✔$$

The solution is (8, 9).

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Writing a Linear System of Equations to Represent a Problem Context

When two or more linear equations define a relationship between quantities, they form a system of linear equations. A system of linear equations can be written and used to solve a problem situation. Once the system is written, determine the most appropriate method for solving the system: graphing, substitution, or linear combinations.

Example

George bought 11 tickets for the spaghetti dinner. Adult tickets cost \$5.25, and child tickets cost \$3.50. The total cost that George paid was \$49. Let a represent the number of adult tickets purchased, and let c represent the number of child tickets purchased.

$$\begin{cases} a + c = 11 \\ 5.25a + 3.50c = 49 \end{cases}$$

$$a + c = 11 \qquad\qquad -5.25(a + c) = -5.25(11) \qquad\qquad -5.25a - 5.25c = -57.75$$

$$5.25a + 3.50c = 49 \qquad\longrightarrow\qquad 5.25a + 3.50c = 49 \qquad\longrightarrow\qquad 5.25a + 3.50c = 49$$

$$\begin{array}{r} -5.25a - 5.25c = -57.75 \\ 5.25a + 3.50c = 49 \\ \hline -1.75c = -8.75 \end{array} \qquad\qquad \begin{aligned} a + c &= 11 \\ a + 5 &= 11 \\ a + 5 - 5 &= 11 - 5 \\ a &= 6 \end{aligned}$$

$$c = 5$$

George purchased 6 adult tickets and 5 child tickets.

Choosing the Best Method to Solve a Linear System

Substitution, linear combinations, and graphing are three methods for determining the solution of a linear system of equations. With substitution, a variable in one equation is substituted with an equivalent expression. Use substitution when a variable in one equation can easily be isolated or the equations are in slope-intercept form. The linear combinations method involves adding two equations together so that they result in an equation in one variable. Use the linear combinations method when the coefficients of like terms are opposites or can be easily made into opposites by multiplication. To use the graphing method, graph both linear equations and locate the point of intersection. The graphing method can be used when the numbers are convenient to graph or if an exact solution is not needed.

Example

Ellen started a small business selling homemade lip balm. She charges $5.50 per tub of lip balm. Ellen spends $100 on supplies for the lip balm in addition to the $1.50 cost per tub. The break-even point can be determined using a system of linear equations.

Substitution method:

$$\begin{cases} y = 5.50x \\ y = 1.50x + 100 \end{cases}$$

$$5.5x = 1.5x + 100$$

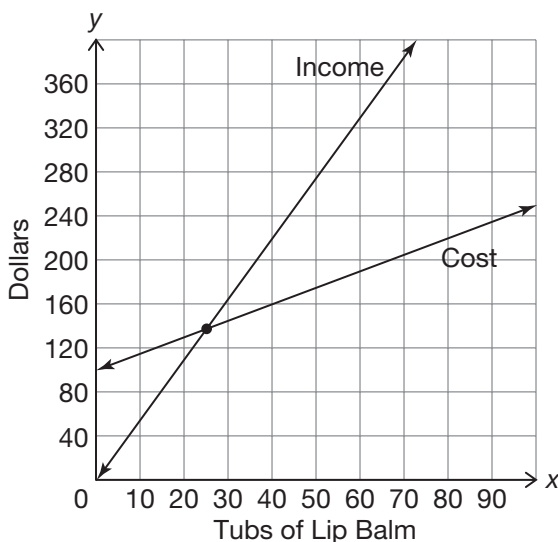$$5.5x - 1.5x = 1.5x - 1.5x + 100$$

$$\frac{4x}{4} = \frac{100}{4} \qquad\qquad y = 5.50(25)$$

$$x = 25 \qquad\qquad y = 137.5$$

The break-even point is (25, 137.5). This means that if Ellen sells 25 tubs of lip balm, her expenses and her income are each $137.50.

Graphing method:

Using the graphing method, the break-even point is approximately (25, 140). If Ellen sells more than 25 tubs of lip balm, her income is more than her expenses. If she sells fewer than 25 tubs of lip balm, her income is less than her expenses.

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