Mathematical Modeling 11

Let's Take a Little Trip

11.1

Every Graph Tells a Story

LEARNING GOALS

In this lesson, you will:

- Identify a linear piecewise function.
- Interpret the graph of a linear piecewise function.
- Determine intervals of increase and decrease for a linear piecewise function.
- Determine values from a graph of a linear piecewise function.
- Physically model the graphs of linear piecewise functions using technology.

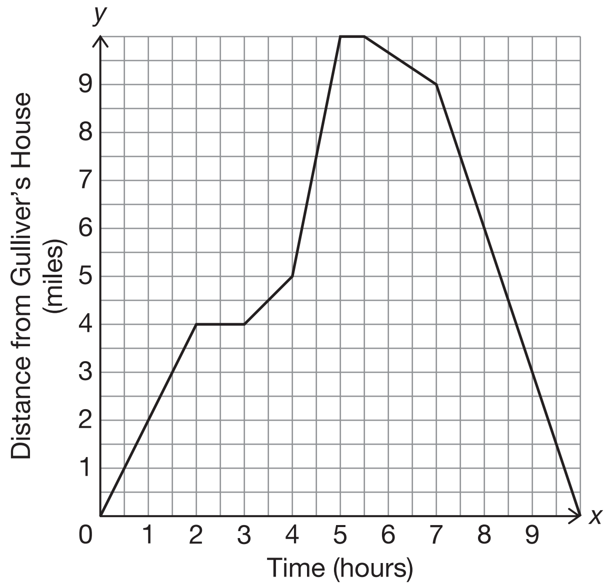
While a person can describe the recent winning streak of a basketball team, or talk about a record number of tornados in certain parts of the country, data graphed on a coordinate plane enables people to *see* data. Graphs relay information about data in a visual way. These same graphs can also be used to make predictions. However, are these predictions accurate?

Often graphs used in textbooks are very organized as are the data sets they are created from. When a graph follows a "perfect" data set, predictions made from it will be accurate—but unfortunately, real life is not so perfect. There are very few examples of real life scenarios in which data sets are perfectly linear, exponential, or quadratic.

Why do you think textbooks use "perfect" data sets? Do you think predictions using real-life data are accurate? Why or why not?

PROBLEM 1 Gulliver's Travels

The graph shows the relation between time in hours and Gulliver's distance from his house during a 10-hour period.



11

Use the graph to answer each question.

1. Is the relation a function? If so, explain why the relation is a function and identify the function family. If not, explain why the relation is not a function.

2. Identify if the relation has any absolute minimum or absolute maximum values. Explain what the absolute minimum or absolute maximum means in terms of this problem situation.

3. Determine the domain and range of this problem situation.

4. How far from home was Gulliver after:

   a. 2 hours?

   b. 2.5 hours?

   c. 6 hours?

   d. 8 hours?

   e. 10 hours?

5. After how many hours was Gulliver:

   a. 5 miles from home?

   b. 10 miles from home?

   c. 4 miles from home?

6. How far did Gulliver travel during the first two hours of the trip?

7. Consider Gulliver's speed during the trip.

   a. What was Gulliver's average speed during the first 2 hours of the trip?

   b. What was Gulliver's average speed between seven and ten hours?

   c. When was Gulliver traveling the fastest? Explain how you know.

11

d. What was Gulliver's average speed when he was traveling the fastest?

e. When was Gulliver traveling the slowest? Explain how you know.

f. What was Gulliver's average speed when he was traveling the slowest?

8. When was Gulliver traveling away from his house? Explain how you know.

9. When was Gulliver traveling toward his house? Explain how you know.

10. When was Gulliver not traveling? Explain how you know.

11. Write a paragraph describing Gulliver's travels.

PROBLEM 2 Acting Out a Linear Piecewise Function

You will need a graphing calculator, a Calculator-Based Ranger (CBR), and a connector cable for this activity. You will also need a meter stick and masking tape to mark off distance measures.

The step-by-step instructions shown tell how to create graphs of linear piecewise functions representing people walking. On these graphs, time can be represented on the x-axis, and distance can be represented on the y-axis. Your goal is to act out each graph by walking in the way that matches the graph. As you act out each graph, your motion will be plotted alongside the graph to monitor your performance.

Step 1: Prepare the workspace.

- Clear an area at least 1 meter wide and 4 meters long leading away from a wall.
- Measure the distances of 0.5, 1, 1.5, 2, 2.5, 3, 3.5, and 4 meters from the wall. Mark these distances on the floor using masking tape.

Step 2: Prepare the technology.

- Connect the CBR to a graphing calculator.
- Transfer the RANGER program from the CBR to the calculator. This only needs to be done the first time. It will then be stored in your calculator.
- Press **2nd LINK** ⟶ **ENTER**.
- Open the CBR and press the appropriate button on it for the type of calculator you are using. Your calculator screen will display **RECEIVING** and then **DONE**. The CBR will flash a green light and beep.

Step 3: Access the RANGER program.

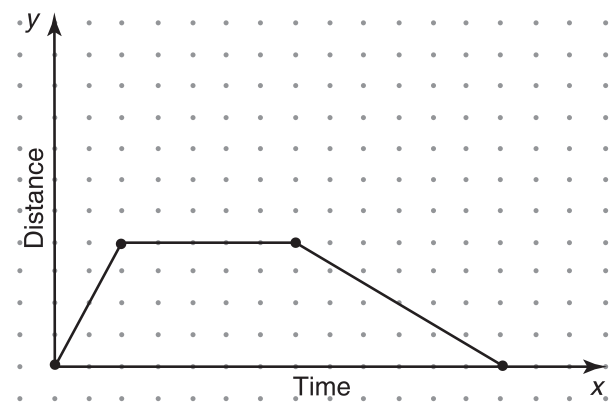
- Press **PRGM** for program. Then scroll to **RANGER**. Press **ENTER**.
- Press **ENTER** to display the **MAIN MENU**.
- Scroll and choose **APPLICATIONS**, and then scoll and choose **METERS**.
- Next, scroll and choose **MATCH** or **DISTANCE MATCH**.
- Press **ENTER**. A graph will be displayed.

Step 4: Act out the graph.

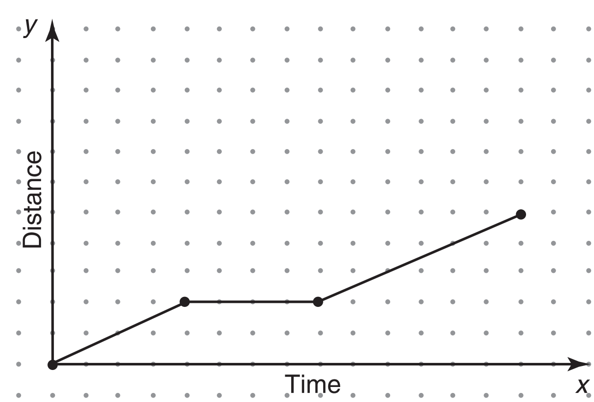
- Examine each graph and plan your path. Use the scale to gauge where to begin in relation to the wall. Will you walk toward or away from the wall? Will you walk fast or slow?
- Hold the graphing calculator in one hand and the CBR in the other hand. The lid of the CBR should be aimed toward the wall.
- Press **ENTER**. Begin walking in a manner that matches the graph. Use the scale and floor markings as guides. You will hear a clicking sound and see a green light as your motion is plotted alongside the piecewise graph on the graphing calculator.
- When the time is finished, examine your performance. What changes should you make?
- Press **ENTER** to display the **OPTIONS** menu. Choose **SAME MATCH**.
- Press **ENTER** and try the walk a second time.
- Continue acting out walks by pressing **ENTER** and **NEW MATCH**.
- When you are finished, press **ENTER** and then scroll and choose **MAIN MENU** and finally **QUIT**.

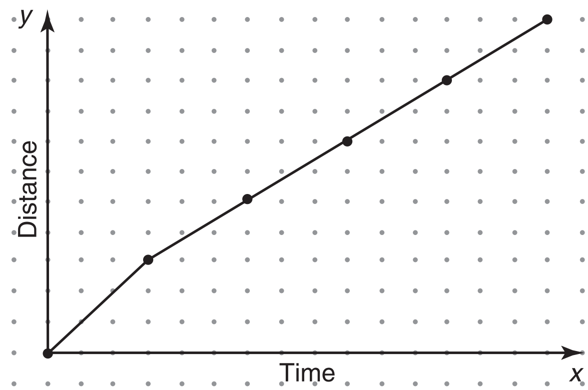
11

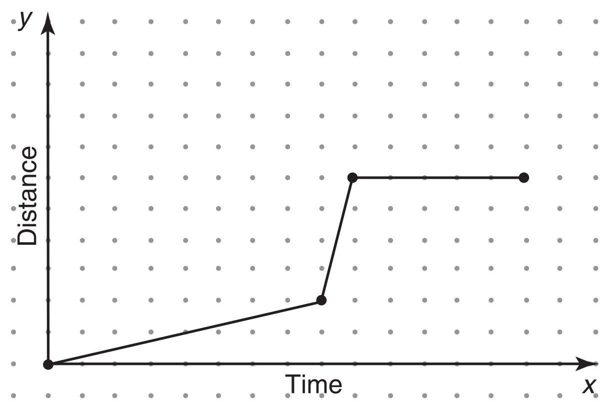
A



B



C


D


1. How did you decide where to stand when beginning to act out a graph?

2. How did you decide when to walk toward the wall and when to back away from the wall?

3. How did you act out a horizontal segment?

4. How did you decide how fast to walk?

5. Sketch your own graph. Then use the CBR to track your motions as you act out the graph. Does the graph you created with the CBR match the graph you sketched?

11

Be prepared to share your solutions and methods.

Whodunit? The Function Family Line-Up

11.2

Modeling Data with Curves of Best Fit

LEARNING GOALS

In this lesson, you will:

- Model data from a scatter plot.
- Identify the function family to which a function belongs.
- Identify graphical behavior of a function.
- Use a model to predict values.
- Interpret parts of a graph.

If you work in technology, Moore's law can be a blessing—and at times, a curse! Moore's law describes a long-term trend in the history of computer hardware that claims that the number of transistors placed on a circuit doubles approximately every two years. What this basically means is that with more transistors comes the ability to make computers faster. The blessing is that technological products like phones and computers will always improve; thus giving the ability to sell new products every two years. The curse: trying to keep up with the technology and the demands of the market. Do you think that Moore's law is sustainable? Or might there be a point when technology can no longer improve its speed to do tasks.

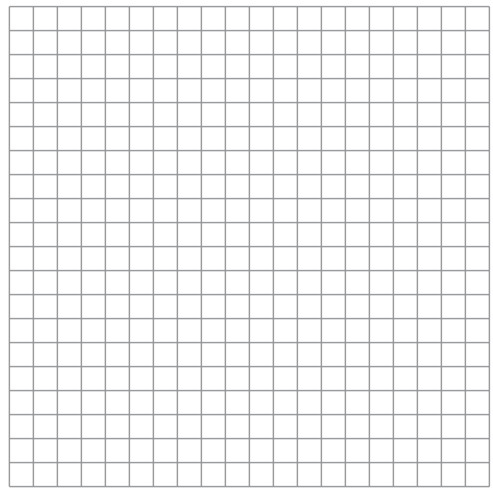
PROBLEM 1 Gas Prices

The table shows the national average annual price of gasoline in the U.S in dollars per gallon for the years 2001 through 2011.

1. Create a scatter plot of the data on the grid shown.

U.S. National Annual Gas Price Average

Year	Dollars per gallon
2001	1.46
2002	1.36
2003	1.59
2004	1.88
2005	2.30
2006	2.59
2007	2.80
2008	3.27
2009	2.35
2010	2.79
2011	3.55



11

2. Sketch a function that best models the data. Then state the function family you sketched.

3. Answer each question about the function you sketched in Question 2.

a. State the domain and range of the function you sketched. How do they compare to the domain and range of this problem situation?

b. Does your function have an absolute minimum, absolute maximum, or neither? If so, describe what it means in terms of this problem situation.

c. Is your function a smooth curve, or is it made up of one or more straight lines?

11

d. Does your function represent continuous or discrete data? Explain your reasoning.

4. Use the function you sketched to answer each question.

   a. According to your function, what was the U.S. national annual average gasoline price per gallon in 2000?

   b. Predict what the U.S. national annual average gasoline price per gallon will be in 2014 using your function.

   c. Predict when the U.S national annual average gasoline price per gallon will be $5.00 using your function.

   d. Predict when the U.S. national average annual gasoline price will reach $10.00 per gallon using your function.

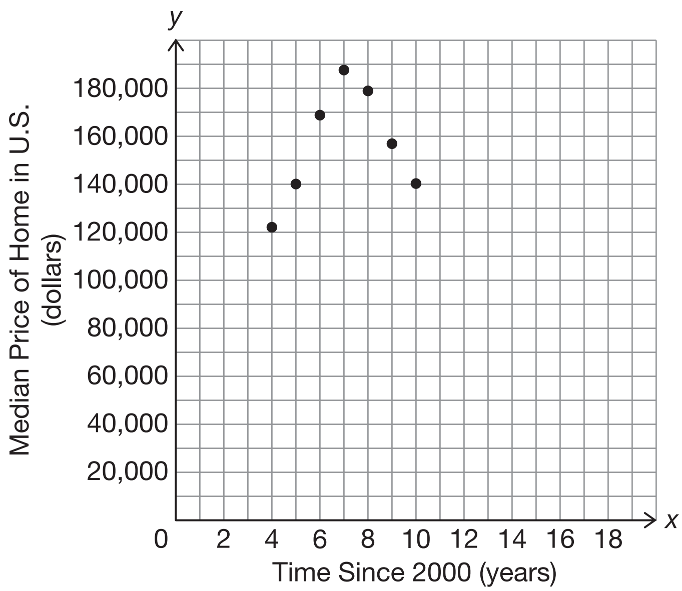
5. Write a brief summary about the national average annual gasoline prices in the U.S. In your summary, you may want to answer the following types of questions: What trends do you see from the data? Is your function a good model for the data, and why or why not? What predictions can you make? Are your predictions realistic? Professionals in what fields may find this type of information useful?

PROBLEM 2 Median Price of a Home

The table shows the median price of a single family home in the U.S for the years 2004 through 2010. A scatter plot of the data is also shown.

U.S. Median Home Price from 2004–2010

Year	Dollars
2004	122,100
2005	140,100
2006	168,800
2007	187,600
2008	178,900
2009	156,900
2010	140,300



1. What do you notice about the median price of a home between the years 2004 and 2010?

2. Sketch a function that best models the data. Then state the function family you sketched.

3. State the domain and range of the function. How do they compare to the domain and range of this problem situation?

4. Estimate the function's *y*-intercept, and interpret its meaning in terms of this problem situation.

5. Does the function have an absolute minimum or maximum? If so, identify it and interpret its meaning in terms of this problem situation. If not, explain why not.

6. Predict the median price of a home in 2001 using your function. Does your prediction make sense in terms of this problem situation? Explain your reasoning.

7. Predict the median home price in 2015 using your function. Does this prediction make sense?

8. What do your answers tell you about the function you chose to model the data?

9. Do some research about the housing market from 2004 to 2010. Use your findings to write a paragraph that justifies the shape of the data and your model.

Be prepared to share your solutions and methods.

People, Tea, and Carbon Dioxide

11.3

Modeling Using Exponential Functions

LEARNING GOALS

In this lesson, you will:

- Write exponential models from data sets.
- Use models to solve problems.

It can be amazing how many different historical events are connected in one way or another. For example, there are some environmentalists who claim that the increase in the world's population has led to an increase in atmospheric gases like carbon dioxide that have led to an overall global warming. In this event, population and environment collide in an issue that is on the minds of many people.

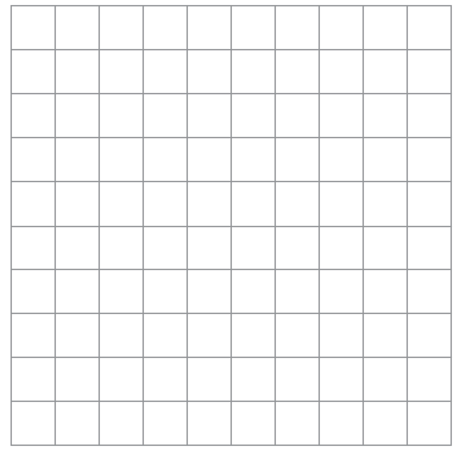
Of course, supporters and opponents of these theories can claim different statistics to support an opinion or contradict a theory. As some say, "facts are facts," but can these facts or statistics be reported in different ways to help support different theories? Do you think that some groups may only report some of the statistics, but not necessarily all of the statistics? Why might they do this?

PROBLEM 1 Populations Keep Growing and Growing and . . .

The table shows the U.S. population in millions of people at 20-year intervals from 1815 to 1975.

U.S. Population 1815–1975

Year	Population (per 1 million people)
1815	8.3
1835	14.7
1855	26.7
1875	44.4
1895	68.9
1915	98.8
1935	127.1
1955	164.0
1975	214.3



11

1. Create a scatter plot of the data on the grid shown. Then use a graphing calculator to determine the exponential regression equation and the value of the correlation coefficient. Finally define, write, and sketch a function to model this situation.

You can use a graphing calculator to determine the exponential regression equation for a data set.

Step 1: Press **STAT** and select **1:Edit.** Enter the data set with the independent variable in **L1** and the dependent variable in **L2**.

Step 2: Press **STAT** and scroll to **CALC.** Then scroll down to **0:ExpReg.** Press **ENTER** twice.

The calculator will display the values of each variable in the form $y = a \cdot b^x$.

Step 3: The r-value displayed represents the correlation coefficient.

2. State the domain and range of this function you sketched. How do they compare to the domain and range of this problem situation?

3. Use your function to predict the population of the United States in 1990.

4. The actual population of the United States in 1990 was 258 million. Compare your predicted population to the actual population. What do you notice? Why do you think this happens?

5. Use your function to predict the population of the United States in 1790.

6. The population actually increased more rapidly after 1790 than what your function predicts. Do some research about U.S. history to explain why this may have happened.

PROBLEM 2 Tea

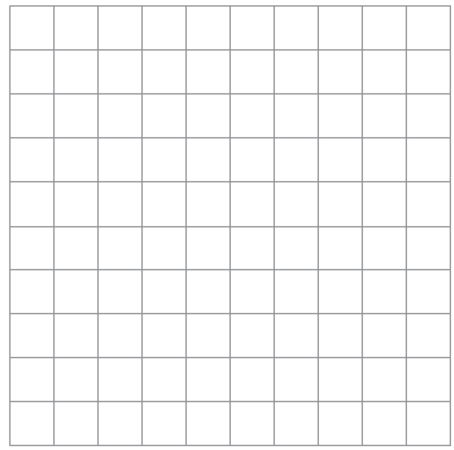
11

Caroline loves green tea after a large meal. After dining out, she boiled water and sat in her kitchen to enjoy her tea.

The table shows the temperature of a cup of Caroline's tea over time.

Temperature of Caroline's Tea Over Time

Time (minutes)	Temperature (degrees Fahrenheit)
0	180
5	169
11	149
15	142
18	135
25	124
30	116
34	113
42	106
45	102
50	101



1. Create a scatter plot of the data on the grid shown. Then use a graphing calculator to determine the exponential regression equation and the value of the correlation coefficient. Finally define, write, and sketch a function to model this situation.

2. State the domain and range of the function you sketched. How do they compare to the domain and range of this problem situation?

3. Use your function to predict the temperature of Caroline's tea after 60 minutes.

4. Use your function to predict when Caroline's tea will reach room temperature, which is 72°F.

11

5. Use your function to predict the temperature of Caroline's tea after 240 minutes.

6. Does your prediction make sense in terms of this problem situation? Explain your reasoning.

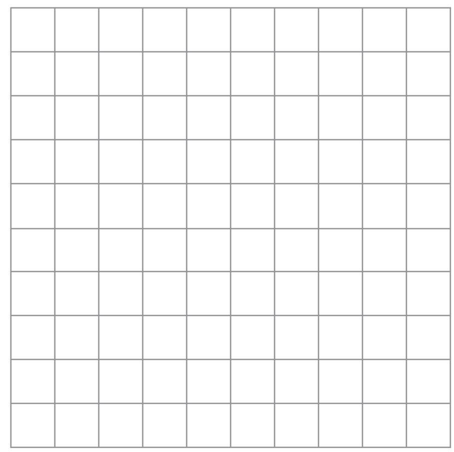
PROBLEM 3 Plants Thrive On It! But Humans Do Not . . .

One measure of climate change is the amount of carbon dioxide in Earth's oceans. When the level of carbon dioxide in the atmosphere increases, the concentration of carbon dioxide in the ocean water also increases.

The table shows the carbon dioxide concentration in the Atlantic Ocean in parts per million from 1750 to 1975.

Carbon Dioxide in the Atlantic Ocean

Year	Carbon Dioxide Concentration (parts per million)
1750	277.0
1775	279.3
1800	282.9
1825	284.3
1850	285.2
1875	289.4
1900	296.7
1925	304.9
1950	312.0
1975	329.4



1. Create a scatter plot of the data on the grid shown. Then use a graphing calculator to determine the exponential regression equation and the value of the correlation coefficient. Finally define, write, and sketch a function to model this situation.

2. State the domain and range of the function you sketched. How do they compare to the domain and range of this problem situation?

3. Use your function to predict the concentration of carbon dioxide in the Atlantic Ocean in the year 2000.

4. Use your function to predict when the concentration of carbon dioxide in the Atlantic Ocean was 250 parts per million.

5. Use your function to predict when the concentration of carbon dioxide in the Atlantic Ocean was 100 parts per million.

6. Using your function, the concentration of carbon dioxide has been increasing exponentially over the past 250 years. What factors could have contributed to this behavior?

Be prepared to share your solutions and methods.

11

BAC Is BAD News

11.4

Choosing the Best Function to Model Data

LEARNING GOALS

In this lesson, you will:

- Determine the type of regression equation that best fits a graph.
- Use a function to model a problem situation.
- Interpret characteristics of a function in terms of a problem situation.
- Analyze results to write a report.

Blood Alcohol Content, or BAC, is a way of measuring the amount of alcohol in a person's blood stream. BAC levels are measured in percentages. A BAC of 0.08 means that 0.08 percent of a person's blood is alcohol. That's not much! In fact, it is a lot less than 1%!

Most states in the United States define a driver that is 21 or older legally impaired at a BAC level of 0.08 or higher. For drivers under 21, *any* BAC level above 0.00 is illegal! So no high school students should ever be drinking alcohol!

PROBLEM 1 Blood Alcohol Content

There is a relationship between the relative probability of a driver causing a car accident and a driver's BAC. A driver with no alcohol in the blood system is defined as having a relative probability of 1 of causing a car accident. A relative probability is the number of times more than 1 that a car accident is likely to occur with alcohol in the blood system. For example, a relative probability of 2 for a driver that has alcohol in the blood system means that a car accident is twice as likely to occur as for a driver with no alcohol in the blood system.

Suppose that a recent study claims that a person with no alcohol in the blood system has a 1.8% chance of causing a car accident.

From this information, you can determine the relative probability of a person causing an accident if that person has any alcohol in the blood system.

Let's suppose a person has a legal limit of 0.06% BAC and that the relative probability of a person with 0.06% BAC causing a car accident is 2.

Use the information of the relative probability of a person with any alcohol in the blood system in comparison with a person who has *no* alcohol in the blood system.

BAC of 0% = 1 (1.8% chance)

BAC of 0.06% = 2

Write an expression to determine the probability of a person causing a car accident with a 0.06% BAC. Remember, the relative probability is the number of times *more* likely a person with alcohol in the blood system will cause an accident than a person with no alcohol in the blood system.

$2(0.018)$

Evaluate the expression to determine the probability.

$2(0.018) = 0.036$

The probability that a person with a BAC of 0.06% will cause a car accident is 3.6%.

1. Use the given information in the worked example to answer each question.

   a. A person with a BAC of 0.10% is just over the legal driving limit. If the person's *relative* probability of causing an accident is 5, what is the probability that this person will cause a car accident?

b. A person with a BAC of 0.16% has twice the BAC of a driver that has just reached the legally impaired mark. If this person's *relative* probability of causing an accident is 25, what is the probability that this person will cause a car accident?

c. What do your answers in parts (a) and (b) tell you about the rate at which alcohol affects a person's ability to drive?

Many people may not realize how quickly a person's BAC can increase. Different factors affect a person's BAC, including weight, gender, the duration of consuming alcohol, and the amount of food the person eats.

According to the Virginia Tech Alcohol Abuse Prevention website, a typical 140-pound male who has one drink over a 40-minute period will have a BAC of 0.03%. If he has another drink over the next 40 minutes, his BAC rises to 0.05%. If he has one more drink, his BAC rises to 0.08%, which means he legally cannot drive.

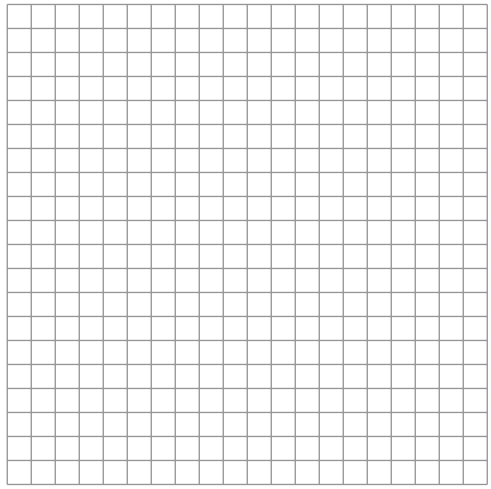
The table shows the results of a study which examined a person's BAC level and the person's relative probability of causing an accident.

BAC Level (percent)	Relative Probability of Causing an Accident (percent)
0.02	1
0.04	1.8
0.06	2
0.08	3
0.10	5
0.12	8
0.14	15
0.16	25
0.18	33

2. Analyze the data in the table. What type(s) of function(s) model this situation? Explain your reasoning.

11

3. Create a scatter plot of the data.



4. Consider the data in the table and your graph. What type(s) of function(s) model this situation based on these two representations? Explain your reasoning.

5. Based on your answer to Question 4, determine the appropriate regression equation. You may use your graphing calculator to determine the equation. Then sketch the function to model this situation.

6. Identify all the important key characteristics of your function in Question 5. Interpret what each means in terms of this problem situation. If applicable, include information about the domain, range, absolute minimums, absolute maximums, intervals of increase, intervals of decrease, x-intercepts, and y-intercepts.

7. Suppose that a driver is within the legal limit with a BAC of 0.075 percent. If a person with no alcohol in the blood system has a 1.8% chance of causing a car accident, predict the probability that a driver with a BAC of 0.075 percent will cause an accident. Do you think that this person is safe to drive, even though the person can legally drive? Explain why or why not. Show all your work.

8. Suppose that a driver has twice the BAC of the driver in Question 7. Predict the probability that this driver will cause an accident.

9. Suppose that a driver has three times the BAC of the driver in Question 7. Predict the probability that this driver will cause an accident.

10. Compare your answers in Question 7 through Question 9. What do you notice? Do they make sense in terms of this problem situation?

11. Write an article for the newsletter of the local chapter of S.A.D.D. (Students Against Destructive Decisions) that stresses the seriousness of drinking and driving. Use the results of this lesson to support your claims. You may want to include facts about the rate at which a driver's probability of causing an accident increases as their BAC increases, the definitions of legal limits in your state, and how a driver's motor skills are affected by alcohol.

Be prepared to share your solutions and methods.

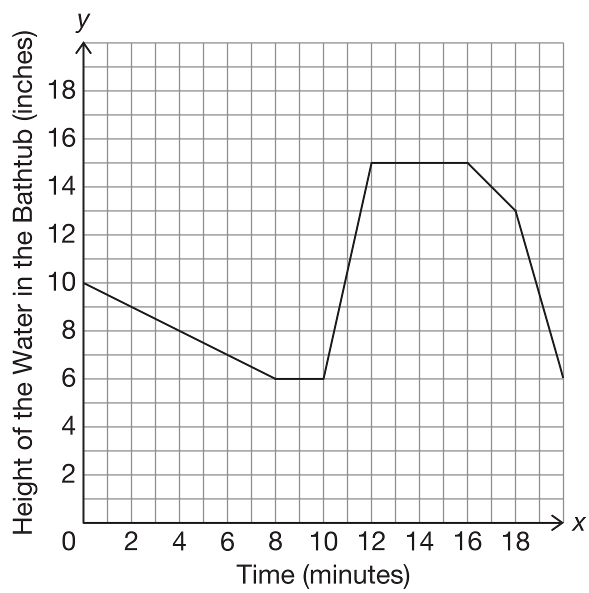
Chapter 11 Summary

11.1 Identifying Characteristics of Linear Piecewise Functions

A linear piecewise function is a piecewise-defined function whose pieces are linear. Characteristics such as domain and range and absolute minimum or absolute maximum can be determined from a piecewise function as well as information regarding the problem scenario.

Example

The graph shown describes a situation that represents the height of water in the bathtub over the 20 minutes that Parker is bathing her dog.



The domain is 0 to 20 minutes. The range is 6 to 15 inches. The absolute maximum is 15 inches, and it occurs between minutes 12 and 16. The absolute minimum is 6 inches, and it occurs between minutes 8 and 10 and at minute 16.

The height of the water decreased slowly between 0 and 8 minutes from 10 inches to 5 inches. It stayed at 5 inches for 2 minutes. The height of water then increased between minutes 10 and 12, when it increased 9 inches. It remained at this height for 4 minutes then slowly decreased for 2 minutes. Finally at 18 minutes, the water began decreasing more quickly.

Parker started her dog's bath with 10 inches of water in the tub. The water was slowly leaking out of the broken drain for the first 8 minutes of the bath. She realized the leak after 8 minutes and placed a washcloth over the drain to stop the leak. After 2 additional minutes, she ran the water for 2 minutes and increased the height of the water by 9 inches. The water held constant until the 16th minute, when the dog kicked the washcloth and slightly opened the drain. At 18 minutes, Parker opened the drain to start letting the water out of the bathtub.

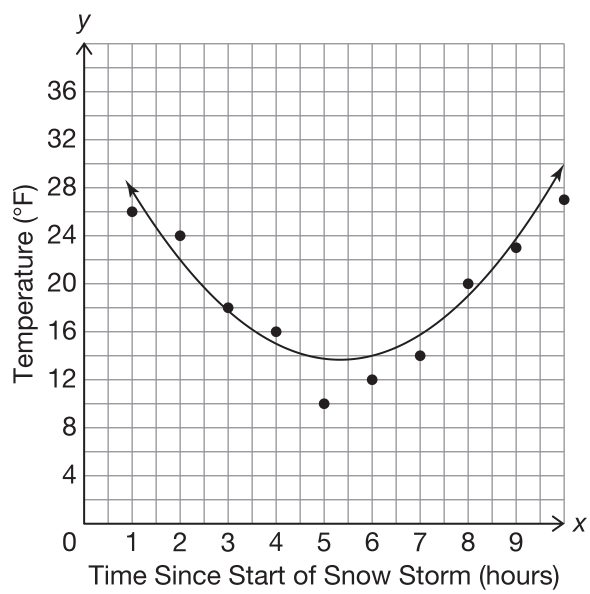
11.2 Modeling Data with Curves of Best Fit

Real life data can be modeled using a scatter plot. Characteristics of the data such as increasing or decreasing over the domain, having an absolute maximum or absolute minimum, and being made of a smooth curve or straight lines can be used to identify the function family to which the function belongs.

Example

The table and graph display temperatures recorded during a 10-hour winter snow storm.

Time Since Start of Snow Storm (hours)	Temperature (°F)
1	26
2	24
3	18
4	16
5	10
6	12
7	14
8	20
9	23
10	27



The function that is used to model the data decreases until about 5 hours and then the function increases.

This function has an absolute minimum at about (5, 14) which means the lowest temperature during the storm occurred about 5 hours after the storm began.

The function is a smooth curve representing continuous data. It is continuous because data exists in the time between hours.

Based on this information, this function belongs to the quadratic function family.

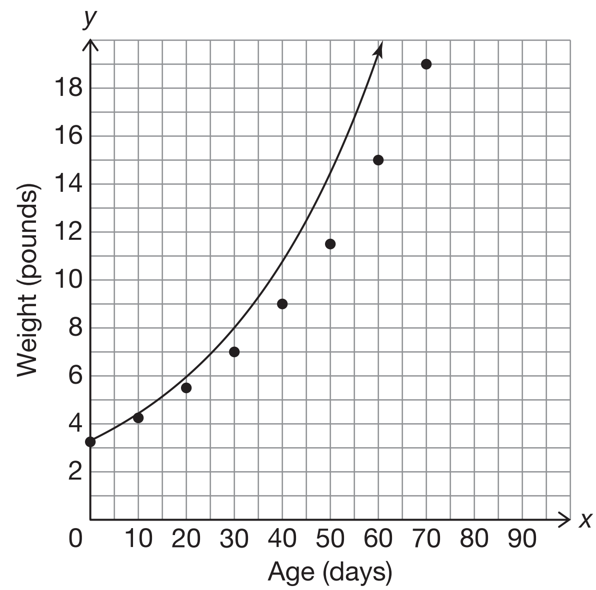
11.3 Writing Exponential Models from Data Sets

A graphing calculator can be used to determine an exponential regression equation to model real-world data. This regression equation can be used to predict future values.

Example

The table shows the weight of a golden retriever puppy as recorded during her growth. A scatter plot and regression equation of the data is shown.

Age (days)	Weight (pounds)
0	3.25
10	4.25
20	5.5
30	7
40	9
50	11.5
60	15
70	19



11

By using a graphing calculator, you can determine that the exponential regression equation for this scenario is $f(x) = 3.3(1.03)^x$. The correlation coefficient, r, is 0.9999. The equation can be used to predict the puppy's weight at 80 days.

$f(80) = 3.3(1.03)^{80}$
$f(80) = 35.1$

The exponential regression equation predicts that the puppy's weight will be approximately 35.1 pounds on day 80.

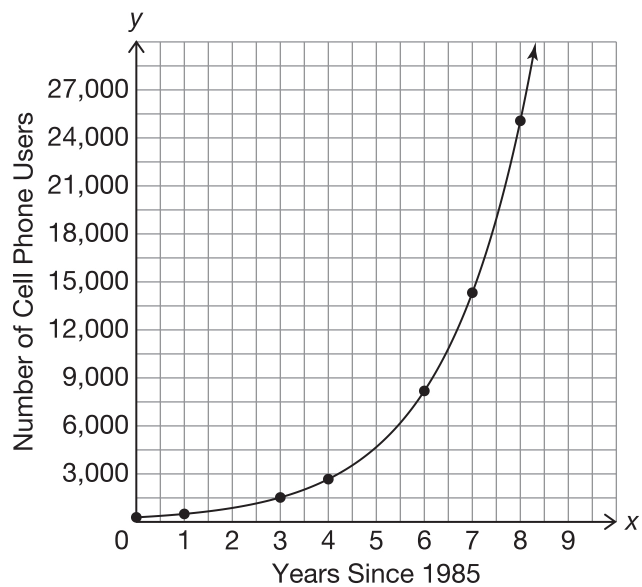
11.4 Determining the Type of Regression Equation that Best Fits a Graph

It is often difficult to tell the type of regression that best fits a data set. Calculating and graphing the regression equation can help determine which type of regression is the best fit.

Example

The cell phone use of Americans has increased dramatically since 1985. The data table shows the number of cell phone subscribers in the small town of Springfield.

Years Since 1985	Number of Cell Phone Users
0	285
1	498
3	1527
4	2672
6	8186
7	14,325
8	25,069



The number of cell phone users increases with the increase of each year. Increasing functions can be modeled by a linear or exponential function. However, because the number of cell phone users does not increase at a constant rate, the data cannot be modeled by a linear function. The exponential regression equation for the data is $f(x) = 285(1.75)^x$. This function when graphed closely models the data.