14 Perimeter and Area of Geometric Figures on the Coordinate Plane

Transforming to a New Level!

14.1

Using Transformations to Determine Perimeter and Area

LEARNING GOALS

In this lesson, you will:

- Determine the perimeter and area of non-square rectangles on a coordinate plane.
- Determine the perimeter and area of squares on a coordinate plane.
- Connect transformations of geometric figures with number sense and operation.
- Determine perimeters and areas of rectangles using transformations.

Did you know that every baseball game has a "seventh-inning stretch" that gives people an opportunity to get up and stretch their legs? While stretching the legs is good for humans—especially at long sporting events—stretching is not helpful when determining the perimeter and area of geometric figures.

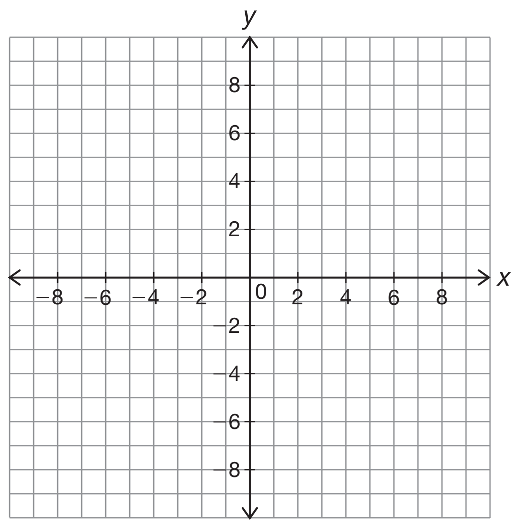
As you learned previously, translations, rotations, and reflections are transformations. And for a transformation to be a transformation, all the points need to be transformed—not just *some* of the points.

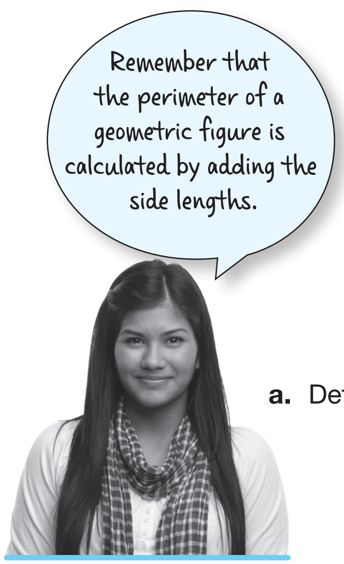
What do you think might happen if you try to translate a figure by moving only one point? What would this "stretching" do to the geometric figure's perimeter and area?

PROBLEM 1 Determining the Outside and Inside

Previously, you determined the congruency of geometric figures. Now, you will determine the perimeter and the area of geometric figures.

1. Graph rectangle *ABCD* with vertices *A*(4, 3), *B*(10, 3), *C*(10, 5), and *D*(4, 5).





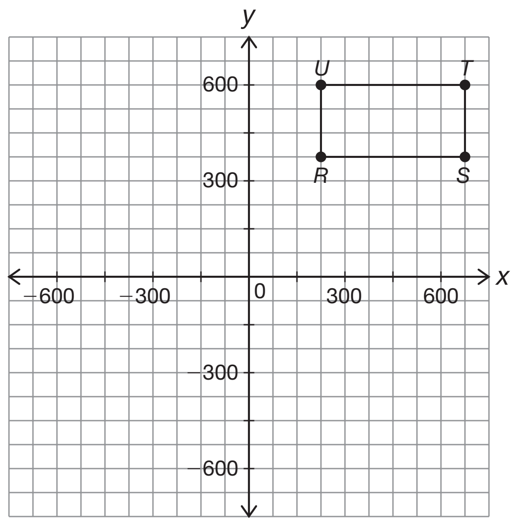
a. Determine the perimeter of rectangle *ABCD*.

b. Determine the area of rectangle *ABCD*.

2. Horace says that he determined the area of rectangle *ABCD* by determining the product *CD*(*CB*). Bernice says that Horace is incorrect because he needs to use the base of the rectangle and that the base is *AB*, not *CD*. Horace responded by saying that *CD* is one of the bases. Who's correct? Explain why the correct student's rationale is correct.

The perimeter or area of a rectangle can be determined efficiently by using the distance formula or by simply counting units on the coordinate plane.

3. Analyze rectangle *RSTU* on the coordinate plane shown.



a. What are the increments for the *x*- and *y*-axes?

b. List some strategies you can use to determine the perimeter and area of rectangle *RSTU*.

c. Determine the coordinates of the vertices of rectangle *RSTU*. Then, calculate the perimeter and area of rectangle *RSTU*.

4. Shantelle claimed she used another strategy to determine the perimeter and area of rectangle *RSTU*. She explained the strategy she used.

Shantelle

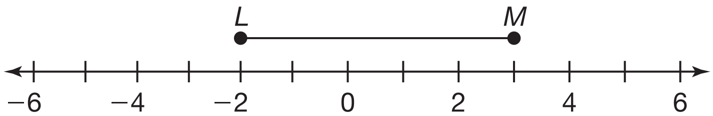
If I translate rectangle RSTU to have at least one point of image R′S′T′U′ on the origin, I can more efficiently calculate the perimeter and area of rectangle RSTU because one of the points will have coordinates (0,0).

Explain why Shantelle's rationale is correct or incorrect.

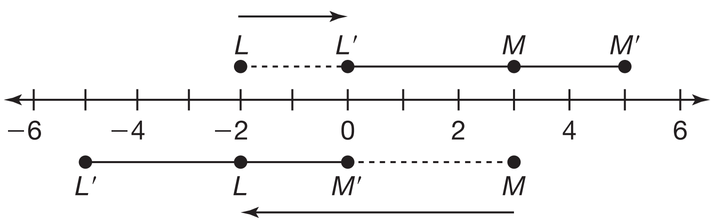
5. If you perform a transformation of rectangle *RSTU* as Shantelle describes, will the image of rectangle *RSTU* have the same area and perimeter as the pre-image *RSTU*? Explain your reasoning.

As you learned previously, transformations are rigid motions that leave a geometric figure unchanged. The pre-image and the image are congruent because in a transformation, *all* vertices must be rigidly moved from one location to another location.

If you are determining the length of a line segment, you can graph the line segment above a number line to determine the length. Segment *LM* is shown.



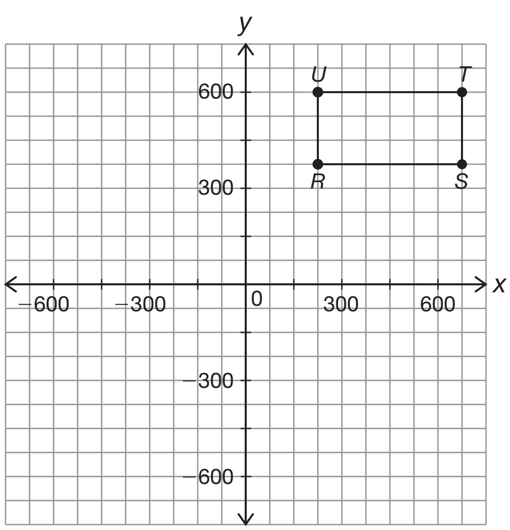
While you can count the number of intervals between −2 and 3, you can also just translate the segment to have the entire segment on the positive side of the number line. Or, you can translate the segment to have the entire segment on the negative side of the number line.



As you can see, segment *LM* and segment *L′M′* are congruent. Thus, the lengths are equal.

So, you know that the lengths of the sides of rectangle *RSTU* will be preserved if the rectangle is translated. That means that the perimeter of the rectangle is preserved when translated.

6. Once again, analyze rectangle *RSTU*.



a. Explain how you can transform the rectangle so that point *R* is located at the origin.

b. Graph rectangle $R'S'T'U'$ on the coordinate plane with point R at the origin. Then, list the coordinates of rectangle $R'S'T'U'$.

c. Identify the points that are *not* at the origin, on the x-axis, or on the y-axis. How do you think this will affect determining the perimeter and area of rectangle $R'S'T'U'$?

d. Determine the perimeter and area of $R'S'T'U'$. What do you notice?

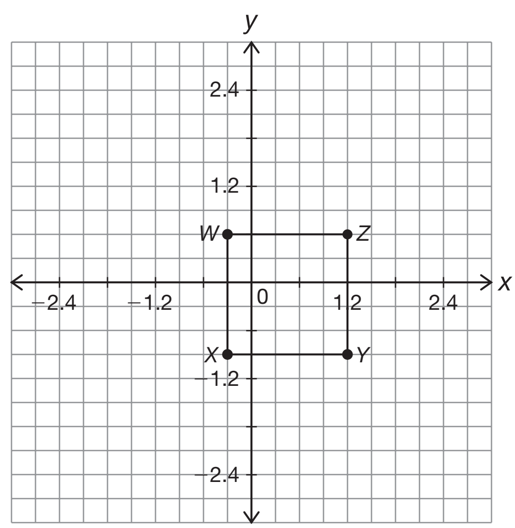
PROBLEM 2 Thinking More About Transformations

You translated a rectangle to allow one vertex to sit at the origin. As a result of the translation, two points also were translated onto the *x*- or *y*-axis, making it possible to use mental calculations to determine the perimeter and area of rectangle *RSTU*.

While making the calculation of perimeter and area more efficient, you actually uncovered yet another way that mathematics maintains balance between different parts of a mathematical problem. Recall that when you use the Distributive Property in a mathematical expression, you must distribute both the value and the operation to all parts of the expression. The same can be said when performing a transformation of a geometric figure.

If a transformation is performed on a geometric figure, not only are the pre-image and the image congruent, but the pre-image's and image's perimeters and areas are equal. Knowing this information will help you make good decisions on how to work more efficiently with geometric figures.

1. Analyze the graph of square *WXYZ* shown on the coordinate plane.



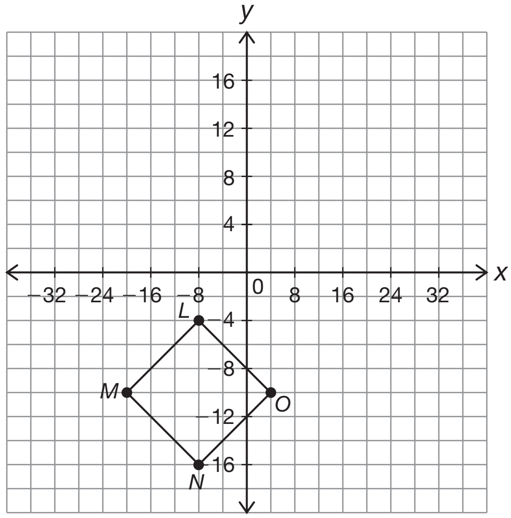
a. Determine the coordinates of square *WXYZ*'s vertices.

b. Do you think that using a transformation could make determining the perimeter and area more efficient? Explain why or why not.

c. Suppose you will perform a transformation to move all the vertices of square *WXYZ* into Quadrant I. Explain which transformation(s) you would perform to determine the perimeter and area in a more efficient way.

d. Determine the perimeter and area of square *WXYZ*.

2. Analyze the graph of the polygon on the coordinate plane shown.



a. Without performing any calculations, predict whether the polygon graphed is a square. Explain how you determined your answer.

b. Determine each vertex coordinate of the polygon. Algebraically verify your prediction. Then determine the perimeter and area.

Be prepared to share your solutions and methods.

Looking at Something Familiar in a New Way

14.2

Area and Perimeter of Triangles on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

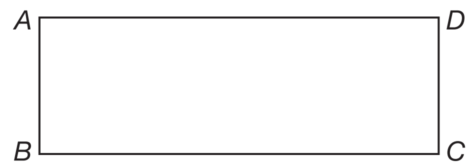
- Determine the perimeter of triangles on the coordinate plane.
- Determine the area of triangles on the coordinate plane.
- Explore the effects doubling the area has on the properties of a triangle.

Your brain can often play tricks on your eyes, and your eyes can play tricks on your brain! Many brain teasers are pictures in which your brain is certain it sees one image; however, by adjusting the way you are looking at the it, suddenly the picture is something totally different! In the early 1990s a series of books was published called *Magic Eye*. These books were autostereograms which are designed to create the illusion of a 3D scene from a 2D image in our brains. When looking at an autostereogram, your eyes send a message to your brain that it is looking at a repeated 2D pattern. However, your eyes are viewing the pattern from slightly different angles so your brain cannot make sense of the pattern. Once you find that correct angle, you can trick your brain into seeing the picture. While many people have fun trying to get the picture to “pop out at them,” eye doctors and vision therapists have actually used these autostereograms in the treatment of some different vision disorders.

Are you able to trick your brain into seeing an image in a different way? When your brain is processing the information differently, is the actual image changing at all? How can you be sure?

PROBLEM 1 Area of a Triangle

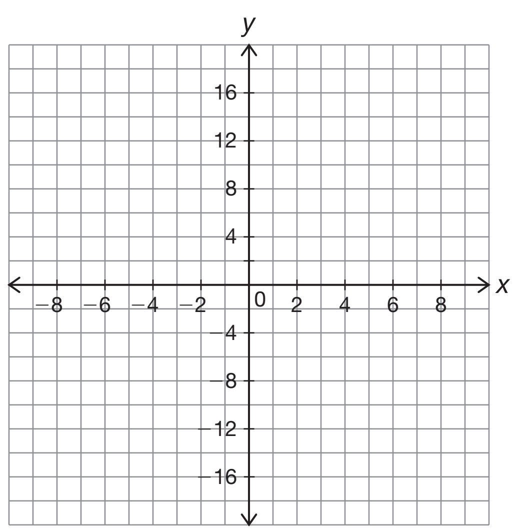
1. The formula for the area of a triangle can be determined from the formula for the area of a rectangle.



   a. Explain how the formula for the area of a triangle is derived using the given rectangle.

   b. Write the formula used to determine the area of a triangle.

2. Graph triangle *ABC* with vertices *A*(−7.5, 2), *B*(−5.5, 13), and *C*(2.5, 2). Then, determine its perimeter.



3. Determine the area of triangle *ABC*.

   a. What information is needed about triangle *ABC* to determine its area?

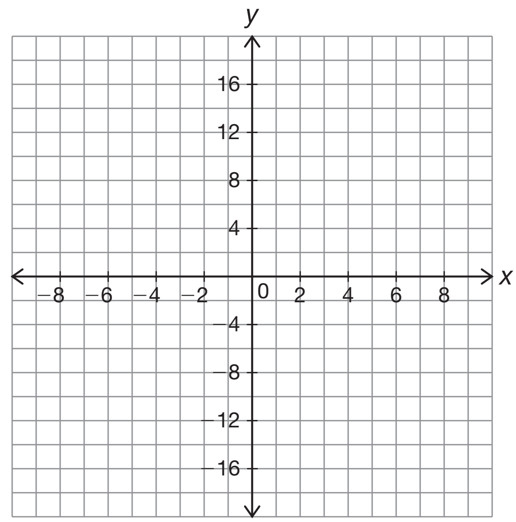
   b. Arlo says that line segment *AB* can be used as the height. Trisha disagrees and says that line segment *BC* can be used as the height. Randy disagrees with both of them and says that none of the line segments currently on the triangle can be used as the height. Who is correct? Explain your reasoning.

   c. Draw a line segment representing the height of triangle *ABC*. Label the segment *BD*. Then, determine the height of triangle *ABC*.

   d. Determine the area of triangle *ABC*.

4. Let's see if there is a more efficient way to determine the area and perimeter of this triangle.

a. Transform triangle *ABC* on the coordinate plane. Label the image *A′B′C′*. Describe the transformation(s) completed and explain your reasoning.



b. Determine the perimeter of triangle *A′B′C′*.

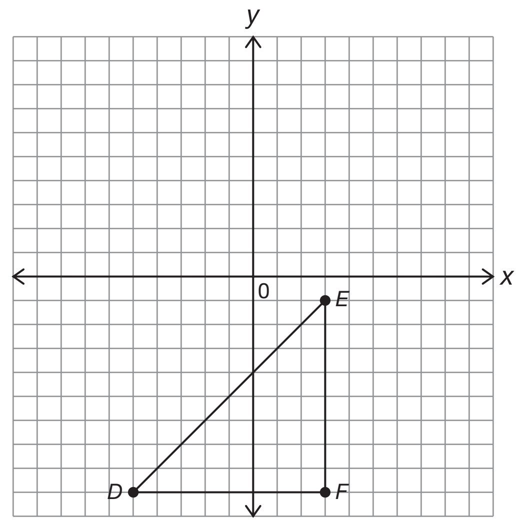
c. Determine the area of triangle $A'B'C'$. Be sure to label the height on the coordinate plane as $B'D'$.

5. Compare the perimeters and areas of triangles ABC and $A'B'C'$.

a. What do you notice about the perimeters and areas of both triangles?

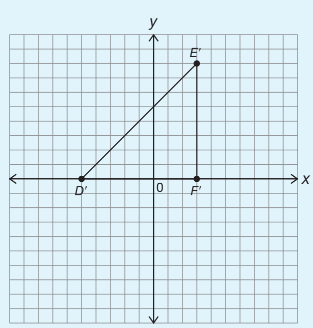
b. Use what you know about transformations to explain why this occurs.

6. Mr. Young gives his class triangle *DEF* and asks them to determine the area and perimeter.

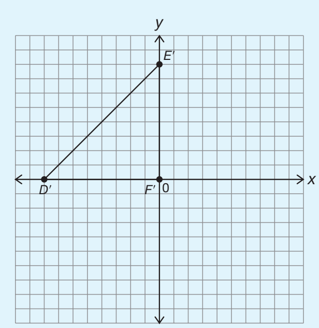


Four of his students decide to first transform the figure and then determine the perimeter and area. Their transformations are shown.

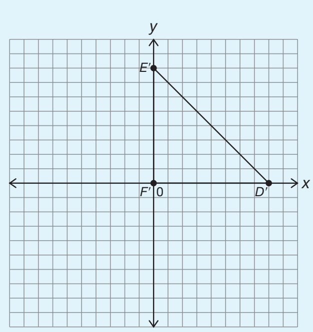
Michael



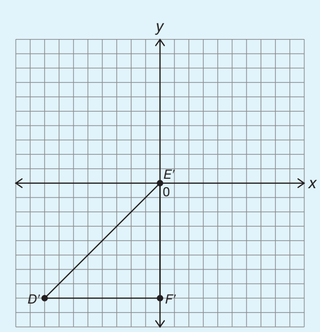
Angelica



Juan



Isabel



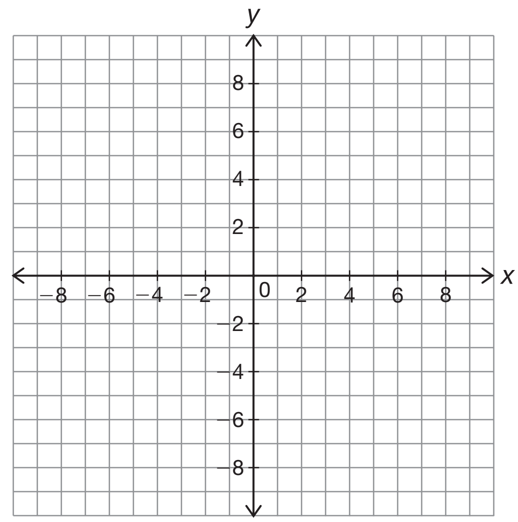
a. Describe the transformation(s) each student made to triangle *DEF*.

b. Whose method is most efficient? Explain your reasoning.

c. What do you know about the perimeter and area of all the triangles? Explain your reasoning.

PROBLEM 2 Which Way Is Up?

1. Graph triangle *ABC* with vertices *A*(2, 5), *B* (10, 9), and *C*(6, 1). Determine the perimeter.



2. To determine the area, you will need to determine the height. How will determining the height of this triangle be different from determining the height of the triangle in Problem 1?

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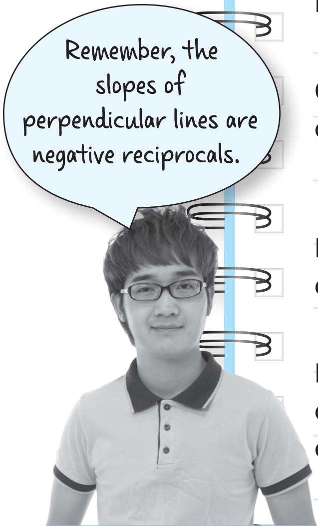
To determine the height of this triangle, you must first understand the relationship between the base and the height. Remember that the height must always be perpendicular to the base.



Let's use *AC* as the base of triangle *ABC*.

Calculate the slope of the base.	$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 5}{6 - 2} = \frac{-4}{4}$ $m = -1$	
Determine the slope of the height.	$m = 1$	
Determine the equation of the base and the equation of the height.	Height *BD* $B(10, 9)$, $m = 1$ $(y - y_1) = m(x - x_1)$ $(y - 9) = 1(x - 10)$ $y = x - 1$	Base *AC* $A(5, 2)$, $m = -1$ $(y - y_1) = m(x - x_1)$ $(y - 2) = -1(x - 5)$ $y = -x + 7$
Solve the system of equations to determine the coordinates of the point of intersection.	$x - 1 = -x + 7$ $2x = 8$ $x = 4$	$y = x - 1$ $y = 4 - 1$ $y = 3$

3. Identify the coordinates of the point of intersection. Graph this point on the coordinate plane and label it point *D*. Draw line segment *BD* to represent the height.

4. Determine the area of triangle *ABC*.

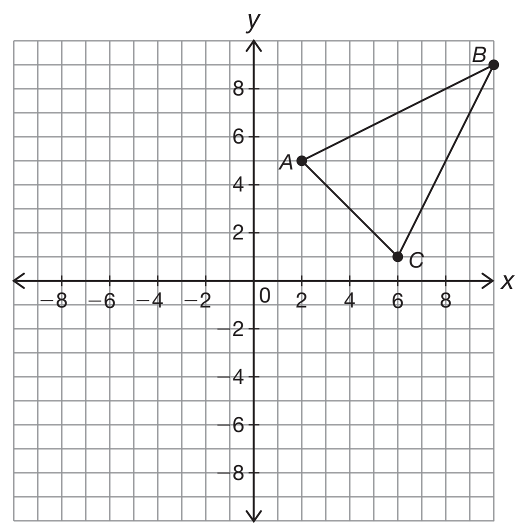
a. Determine the height of the triangle.

b. Determine the area of the triangle.

5. You know that any side of a triangle can be thought of as the base of the triangle. Predict whether using a different side as the base will result in a different area of the triangle. Explain your reasoning.

Let's consider your prediction.

6. Triangle *ABC* is given on the coordinate plane. This time let's consider side *AB* as the base.



a. Let point *D* represent the intersection point of the height, *CD*, and the base. Determine the coordinates of point *D*.

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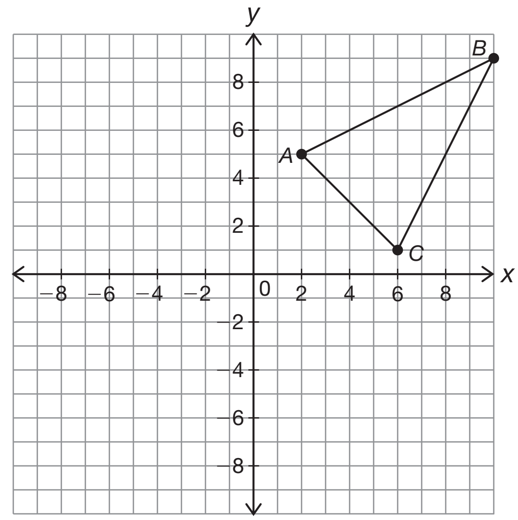
b. Determine the height of triangle *ABC*.

c. Determine the area of triangle *ABC*.

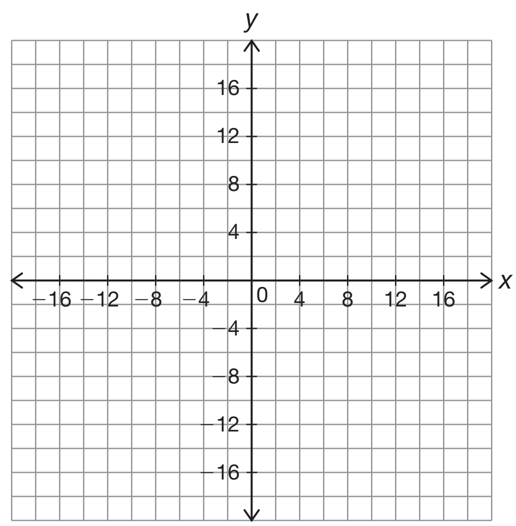
7. Triangle *ABC* is graphed on the coordinate plane. Determine the area of triangle *ABC* using *BC* as the base.



8. Compare the three areas you determined for triangle *ABC*. Was your prediction in Question 5 correct?

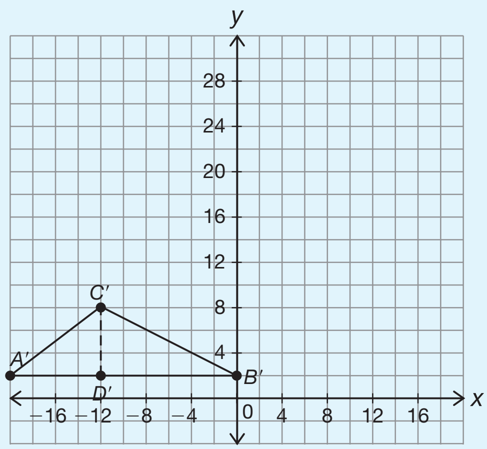
PROBLEM 3 Double Trouble

1. Graph triangle *ABC* with vertices *A*(−14, 2), *B*(−4, 2), and *C*(−12, 5). Determine the area.



2. Brandon's teacher asks his class to double the area of triangle *ABC* by manipulating the pre-image. Brandon's work is shown.

Brandon



I determined the length of the base of the pre-image as 10 units long. In the image, I doubled it so the base, $A'B'$, is now 20 units long.

I then determined the height of the pre-image as 3 units. In the image, I doubled it so the height, $C'D'$, is now 6 units. This doubled the area of triangle *ABC*.

a. Determine the area of Brandon's triangle. How does this relate to the area of the pre-image?

b. Describe Brandon's error and what he should do to draw a triangle that is double the area of the pre-image.

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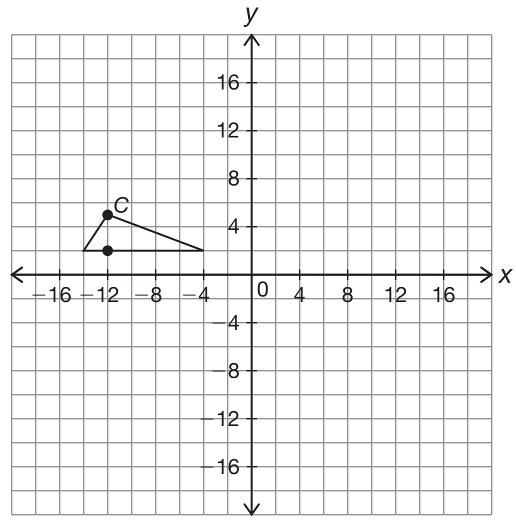
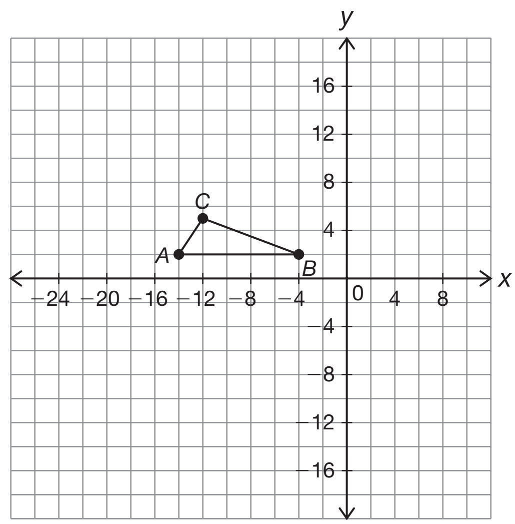
3. Triangle *ABC* is given. Double the area of triangle *ABC* by manipulating the height. Label the image *ABC'*.



a. Paul identified the coordinates of point *C'* as (−12, 8). Olivia disagrees with him and identifies the coordinates of point *C'* as (−12, −4). Who is correct? Explain your reasoning.

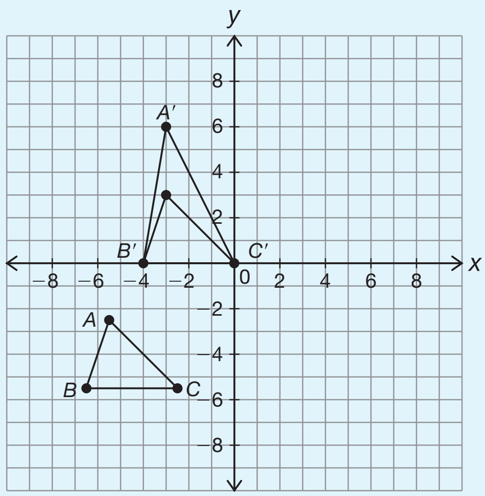
b. Determine the area of triangle *ABC'*.

4. Triangle *ABC* is given. Double the area of triangle *ABC* by manipulating the base. Label the image, identify the coordinates of the new point, and determine the area.



5. Emilio's class is given triangle *ABC*. They are asked to double the area of this triangle by manipulating the height. They must identify the coordinates of the new point, *A′*, and then determine the area. Emilio decides to first translate the triangle so it sits on grid lines to make his calculations more efficient. His work is shown.

Emilio



The coordinates of point *A′* are (-3, 6).

The area of triangle *A′B′C′* is 12 square units.

$$A = \frac{1}{2}(4)(6)$$

$$A = 12$$

Emilio is shocked to learn that he got this answer wrong. Explain to Emilio what he did wrong. Determine the correct answer for this question.

Be prepared to share your solutions and methods.

One Figure, Many Names

14.3

Area and Perimeter of Parallelograms on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

- Determine the perimeter of parallelograms on a coordinate plane.
- Determine the area of parallelograms on a coordinate plane.
- Explore the effects doubling the area has on the properties of a parallelogram.

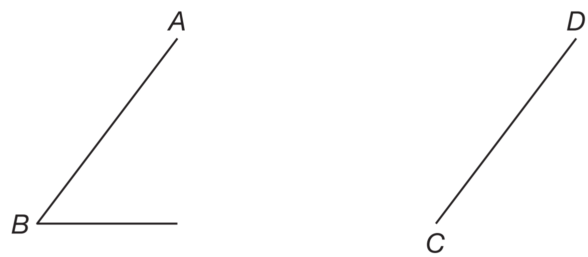
A parallelogram is a quadrilateral with two pairs of parallel sides. However, this is a pretty simple definition when it comes to the many figures we work with in geometry. There are actually different types of parallelograms depending on special features it may have. A parallelogram with four angles of equal measure is known as a rectangle. A parallelogram with four sides of equal length is known as a rhombus. A parallelogram with four sides of equal length and four angles of equal measure is a . . . well, you should know that one.

While working with the parallelograms in the following lesson, try to determine whether they are a certain type of parallelogram. How might knowing a parallelogram is a rectangle or square help you when determining the area or perimeter? But remember, you should never assume a figure has certain measures unless you can prove it!

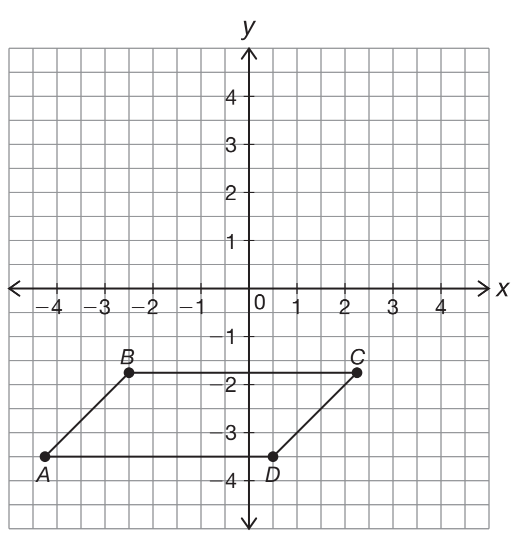
PROBLEM 1 Rectangle or Parallelogram?

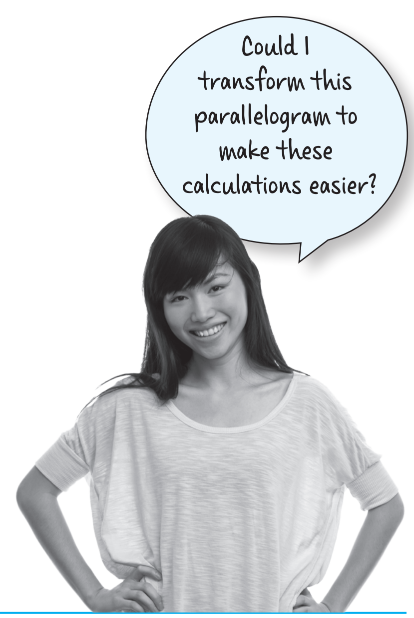
You know the formula for the area of a parallelogram. The formula, $A = bh$, where A represents the area, b represents the length of the base, and h represents the height is the same formula that is used when determining the area of a rectangle.
But how can that be if they are different shapes?

1. Use the given parallelogram to explain how the formula for the area of a parallelogram and the area of a rectangle can be the same.



2. Analyze parallelogram *ABCD* on the coordinate plane.





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a. Determine the perimeter of parallelogram *ABCD*.

b. To determine the area of *ABCD*, you must first determine the height. Describe how to determine the height of *ABCD*.

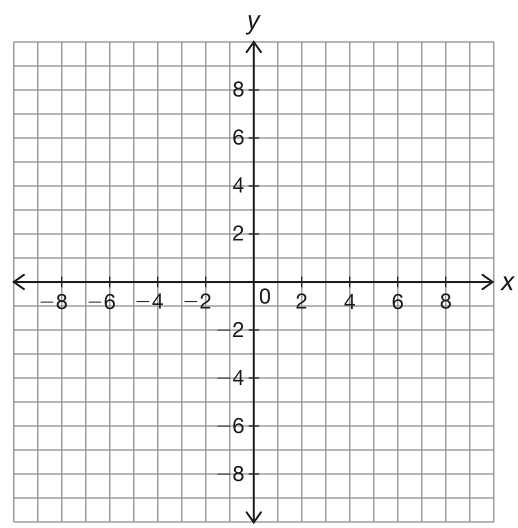
c. Ms. Finch asks her class to identify the height of *ABCD*. Peter draws a perpendicular line from point *B* to line segment *AD*, stating that the height is represented by line segment *BE*. Tonya disagrees. She draws a perpendicular line from point *B* to line segment *BC*, stating that the height is represented by line segment *DF*. Who is correct? Explain your reasoning.

d. Determine the height of *ABCD*.

e. Determine the area of *ABCD*.

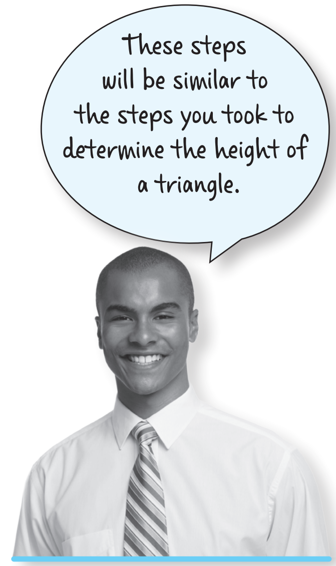
PROBLEM 2 Stand Up Straight!

1. Graph parallelogram *ABCD* with vertices *A*(1, 1), *B*(7, −7), *C*(8, 0), and *D*(2, 8). Determine the perimeter.



2. Determine the area of parallelogram *ABCD*.

   a. Using *CD* as the base, how will determining the height of this parallelogram be different from determining the height of the parallelogram in Problem 1?



   b. Using *CD* as the base, explain how you will locate the coordinates of point *E*, the point where the base and height intersect.

   c. Determine the coordinates of point *E*. Label point *E* on the coordinate plane.

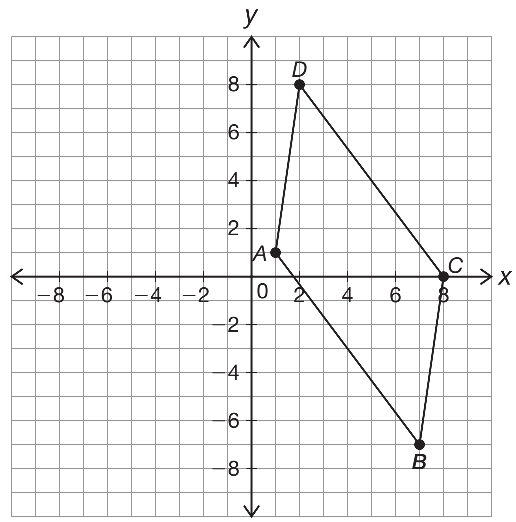
d. Determine the height of parallelogram *ABCD*.

e. Determine the area of parallelogram *ABCD*.

3. You determined earlier that any side of a parallelogram can be thought of as the base. Predict whether using a different side as the base will result in a different area of the parallelogram. Explain your reasoning.

Let's consider your prediction.

4. Parallelogram *ABCD* is given on the coordinate plane. This time let's consider side *BC* as the base.



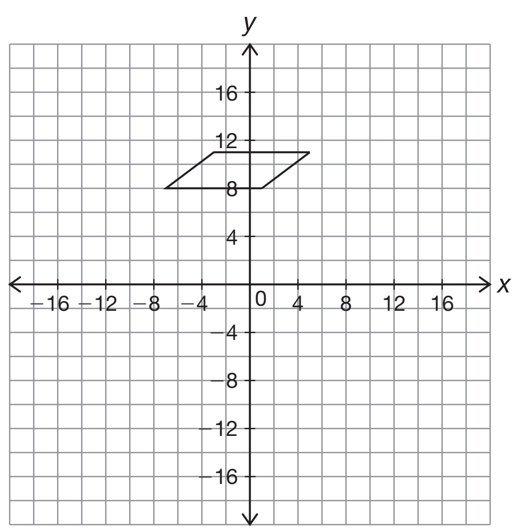
a. Let point *E* represent the intersection point of the height, *AE*, and the base. Determine the coordinates of point *E*.

b. Determine the area of parallelogram *ABCD*.

5. Compare the area you calculated in Question 4, part (b) with the area you calculated in Question 2, part (e). Was your prediction in Question 3 correct?

PROBLEM 3 Double Trouble

1. Graph parallelogram *ABCD* with the vertices $A(-7, 8)$, $B(-3, 11)$, $C(5, 11)$, and $D(1, 8)$.



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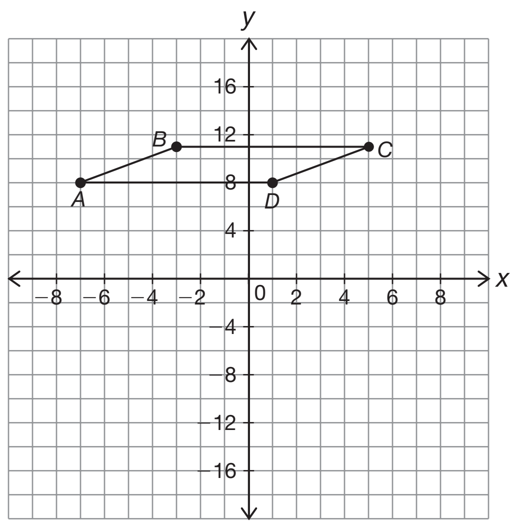
a. Determine the area of parallelogram *ABCD*.

b. You want to double the area of parallelogram *ABCD* by manipulating the base. Describe how you would move the points to represent this on the coordinate plane.

c. Manipulate parallelogram *ABCD* on the coordinate plane as you described above to double the area. Label the image and identify the coordinates of the new point(s).

d. Determine the area of the manipulated triangle.

2. Parallelogram *ABCD* is given. Double the area of parallelogram *ABCD* by manipulating the height. Label the image, identify the coordinates of the new point(s), and determine the area.



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Be prepared to share your solutions and methods.

Let's Go Halfsies!

14.4

Determining the Perimeter and Area of Trapezoids and Composite Figures

LEARNING GOALS

In this lesson, you will:

- Determine the perimeter and area of trapezoids and hexagons on a coordinate plane.
- Use composite figures to determine the perimeter on a coordinate plane.

KEY TERMS

- bases of a trapezoid
- legs of a trapezoid
- regular polygon
- composite figure

You've probably heard it before: "let's go halfsies" when you and a friend of yours want something, but there is only one left. It can even be more annoying if your guardian tells you that you have to "go halfsies" with your sibling!

Of course, for some reason, when you split a bill, the term is *not* "halfsies," but this is called "going dutch." Wow! This can get confusing!

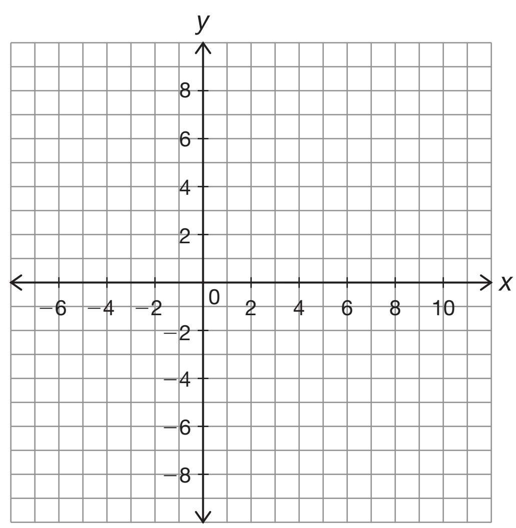
So, what area formulas seem to go "halfsies" with another area formula. Here's a hint: it might have to do something with triangles!

PROBLEM 1 Well, It's the Same, But It's Also Different!

So far, you have determined the perimeter and area of rectangles, squares, and parallelograms. However, there is one last quadrilateral you will discover and you will determine its perimeter and area. Can you name the mysterious quadrilateral?

1. Plot each point on the coordinate plane shown:
   - A (−5, 4)
   - B (−5, −4)
   - C (6, −4)
   - D (0, 4)

   Then, connect the points in alphabetical order.



2. What quadrilateral did you graph? Explain how you know.

The final quadrilateral you will work with is the trapezoid. The trapezoid is unique in the quadrilateral family because it is a quadrilateral that has *exactly* one pair of parallel sides. The parallel sides are known as the **bases of the trapezoid**, while the non-parallel sides are called the **legs of the trapezoid**.

3. Using the trapezoid you graphed, identify:

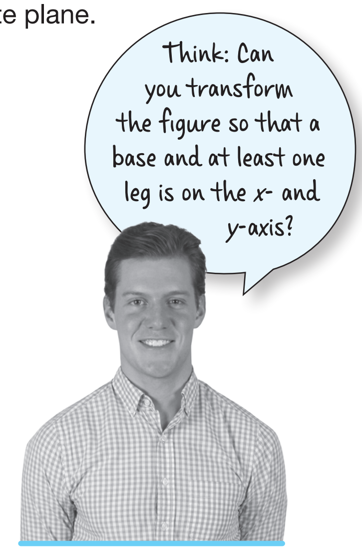
a. the bases.

b. the legs.

Like the other quadrilaterals in this chapter, you can use various methods to determine the perimeter and area of a trapezoid.

4. Analyze trapezoid *ABCD* that you graphed on the coordinate plane.

a. Describe a way that you can determine the perimeter of trapezoid *ABCD* without using the distance formula.



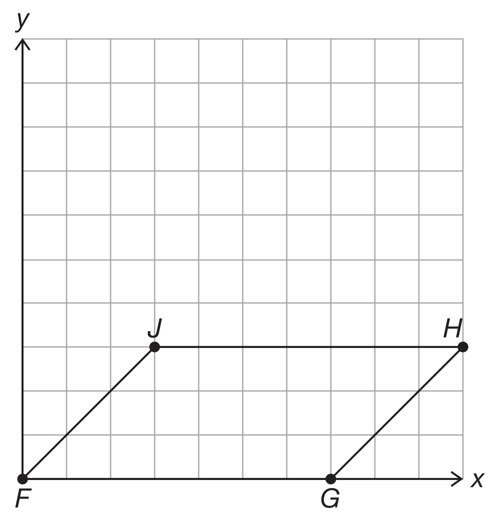
b. Determine the perimeter of trapezoid *ABCD* using the strategy you described in part (a). First, perform a transformation of trapezoid *ABCD* on the coordinate plane and then calculate the perimeter of the image.

PROBLEM 2 Using What You Know

In the last lesson, you learned how to calculate the area of parallelograms. You can use this knowledge to calculate the area of a trapezoid.

So, what similarities exist when determining the area of a parallelogram and a trapezoid?

1. Analyze parallelogram *FGHJ* on the coordinate plane.



Recall that the formula for the area of a parallelogram is $A = bh$, where b represents the base and h represents the height. As you know, a parallelogram has both pairs of opposite sides parallel. But what happens if you divide the parallelogram into two congruent geometric figures?

a. Divide parallelogram *FGHJ* into two congruent geometric figures.

b. What do you notice about parallelogram *FGHJ* when it is divided into two congruent geometric figures?

c. Label the two vertices that make up the two congruent geometric figures.

d. Label the pair of bases that are congruent to each other.
Label one pair of bases b_1 and the other pair b_2.

e. Now write a formula for the area of the entire geometric figure. Make sure you use the bases you labeled and do not forget the height.

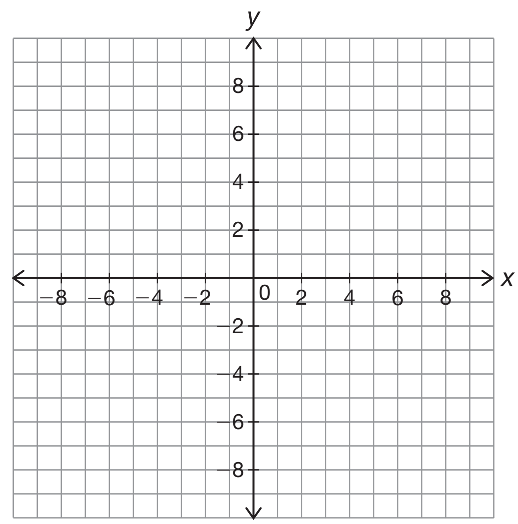
f. Now write the formula for the area for *half* of the entire figure.

2. What can you conclude about the area formula of a parallelogram and the area formula of a trapezoid? Why do you think this connection exists?

3. Plot each point on the coordinate plane shown:

- Q (−2, −2)
- R (5, −2)
- S (5, 2)
- T (1, 2)

Then, connect the points in alphabetical order.



4. Determine the area of trapezoid $QRST$. Describe the strategy or strategies you used to determine your answer.

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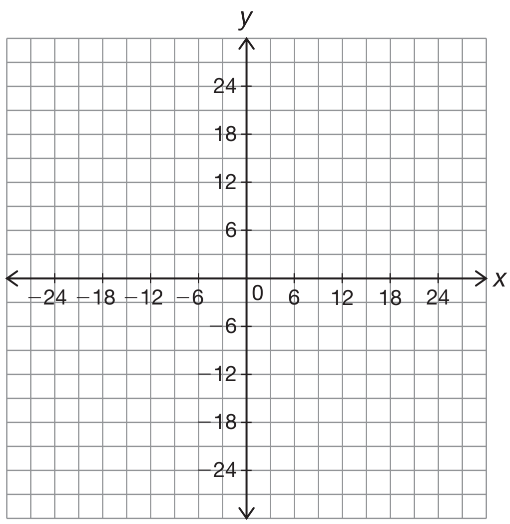
PROBLEM 3 Regular Or Composite?

Now that you have determined the perimeters and areas of various quadrilaterals, you can use this knowledge to expand your ability to determine the perimeter and area of *regular polygons* and *composite figures*. A **regular polygon** is a polygon whose sides all have the same length and whose angles all have the same measure. A **composite figure** is a figure that is formed by combining different shapes.

1. Emma plots the following six points to create the polygon shown on the coordinate plane:

$A(-6, 20)$, $B(-21, 12)$, $C(-21, -5)$, $D(-6, -13)$, $E(9, -5)$, and $F(9, 12)$.



After analyzing the figure, she says that this polygon is a regular hexagon because all the sides are equal. Kevin disagrees and reminds her that she must measure the angles before she can say it is regular. Emma replies that if the side lengths are equal the angles must be equal. Who is correct?

a. Determine the length of each side of the hexagon.

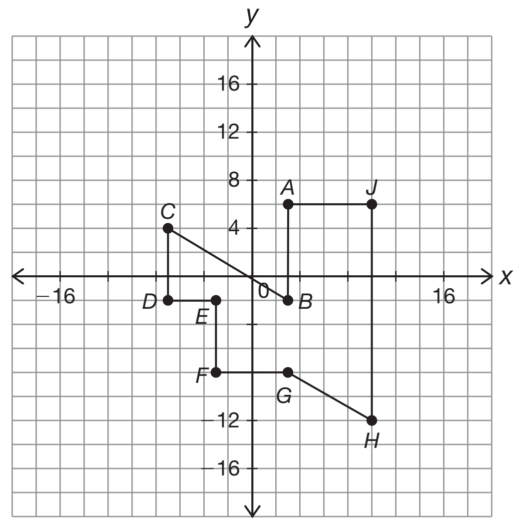
b. Use a protractor to determine the measure of each angle.

c. Use your answers to parts (a) and (b) to tell who is correct. Explain your reasoning.

2. Determine the perimeter of hexagon *ABCDEF*.

3. Determine the area of hexagon *ABCDEF*. Describe how you determined the area and show your work.

4. A composite figure is graphed on the coordinate plane shown.

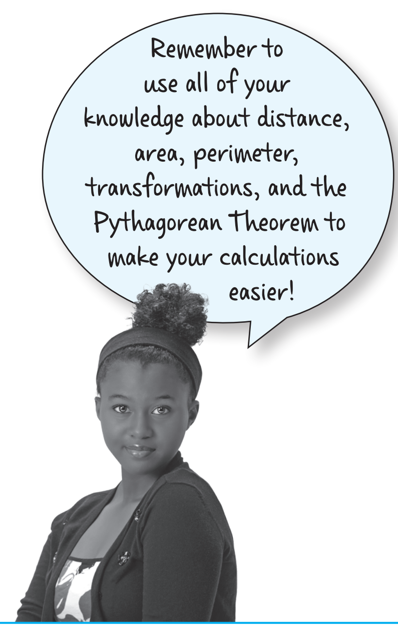


a. Draw line segments on the figure to divide it into familiar shapes.

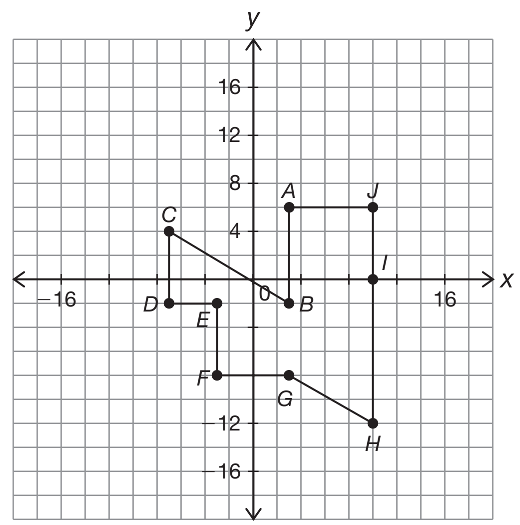
b. What shape(s) did you divide the composite figure into?

c. Determine the perimeter of the composite figure. Round to the nearest tenth if necessary.

d. Determine the area of the composite figure. Round to the nearest tenth if necessary.



5. Draw line segments on the composite figure to divide the figure differently from how you divided it in Question 4.



a. What shapes did you divide the composite figure into?

b. Determine the area of the composite figure. Round to the nearest tenth if necessary.

c. How does the area in Question 4 part (d) compare to the area in part (b)? Explain your reasoning.

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Be prepared to share your solutions and methods.

Chapter 14 Summary

KEY TERMS

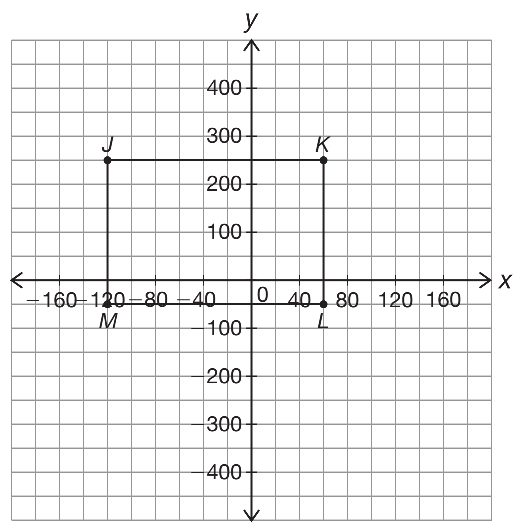
- bases of a trapezoid (14.4)
- legs of a trapezoid (14.4)
- regular polygon (14.4)
- composite figure (14.4)

14.1 Determining the Perimeter and Area of Rectangles and Squares on the Coordinate Plane

The perimeter or area of a rectangle can be calculated using the distance formula or by counting the units of the figure on the coordinate plane. When using the counting method, the units of the *x*-axis and *y*-axis must be considered to count accurately.

Example

Determine the perimeter and area of rectangle *JKLM*.



The coordinates for the vertices of rectangle *JKLM* are *J*(−120, 250), *K*(60, 250), *L*(60, −50), and *M*(−120, −50).

Because the sides of the rectangle lie on grid lines, subtraction can be used to determine the lengths.

$$JK = 60 - (-120) = 180$$

$$KL = 250 - (-50) = 300$$

$$A = bh = 180(300) = 54{,}000$$

The area of rectangle *JKLM* is 54,000 square units.

$$P = JK + KL + LM + JM = 180 + 300 + 180 + 300 = 960$$

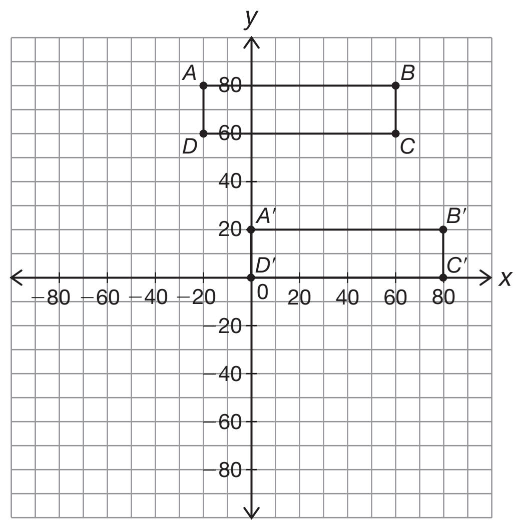
The perimeter of rectangle *JKLM* is 960 units.

14.1 Using Transformations to Determine the Perimeter and Area of Rectangles and Squares

If a rigid motion is performed on a geometric figure, not only are the pre-image and the image congruent, but both the perimeter and area of the pre-image and the image are equal. Knowing this makes solving problems with geometric figures more efficient.

Example

Determine the perimeter and area of rectangle *ABCD*.



The vertices of rectangle *ABCD* are *A*(−20, 80), *B*(60, 80), *C*(60, 60), and *D*(−20, 60). To translate point *D* to the origin, translate *ABCD* to the right 20 units and down 60 units. The vertices of rectangle *A′B′C′D′* are *A′*(0, 20), *B′*(80, 20), *C′*(80, 0), and *D′*(0, 0).

Because the sides of the rectangle lie on grid lines, subtraction can be used to determine the lengths.

$$A'D' = 20 - 0 = 20 \qquad C'D' = 80 - 0 = 80$$

$$P = A'B' + B'C' + C'D' + A'D' = 80 + 20 + 80 + 20 = 200$$

The perimeter of rectangle *A′B′C′D′* and, therefore, the perimeter of rectangle *ABCD*, is 200 units.

$$A = bh = 20(80) = 1600$$

The area of rectangle *A′B′C′D′* and, therefore, the area of rectangle *ABCD*, is 1600 square units.

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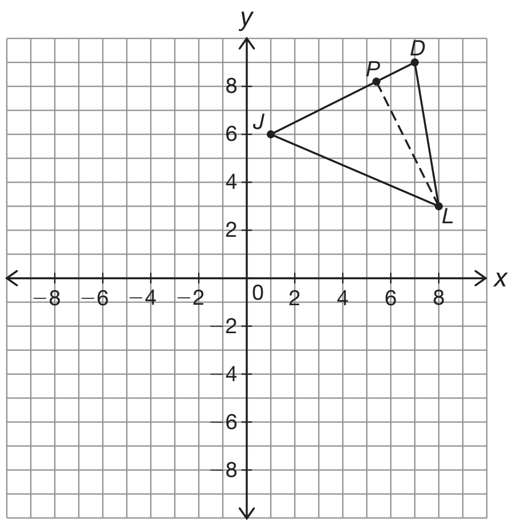
14.2 Determining the Perimeter and Area of Triangles on the Coordinate Plane

The formula for the area of a triangle is half the area of a rectangle. Therefore, the area of a triangle can be found by taking half of the product of the base and the height. The height of a triangle must always be perpendicular to the base. On the coordinate plane, the slope of the height is the negative reciprocal of the slope of the base.

Example

Determine the perimeter and area of triangle *JDL*.



The vertices of triangle *JDL* are $J(1, 6)$, $D(7, 9)$, and $L(8, 3)$.

$$JD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(7 - 1)^2 + (9 - 6)^2} = \sqrt{6^2 + 3^2} = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$$

$$DL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(8 - 7)^2 + (3 - 9)^2} = \sqrt{1^2 + (-6)^2} = \sqrt{1 + 36} = \sqrt{37}$$

$$LJ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1 - 8)^2 + (6 - 3)^2} = \sqrt{(-7)^2 + 3^2} = \sqrt{49 + 9} = \sqrt{58}$$

$$P = JD + DL + LJ = 3\sqrt{5} + \sqrt{37} + \sqrt{58} \approx 20.4$$

The perimeter of triangle *JDL* is approximately 20.4 units.

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To determine the area of the triangle, first determine the height of triangle *JDL*.

Slope of *JD*: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{9 - 6}{7 - 1}$

$= \frac{3}{6}$

$= \frac{1}{2}$

Slope of *PL*: $m = -2$

Equation of *JD*: $(y - y_1) = m(x - x_1)$

$y - 6 = \frac{1}{2}(x - 1)$

$y = \frac{1}{2}x + 5\frac{1}{2}$

Equation of *PL*: $(y - y_1) = m(x - x_1)$

$y - 3 = -2(x - 8)$

$y = -2x + 19$

Intersection of *JD* and *PL*, or *P*: $\frac{1}{2}x + 5\frac{1}{2} = -2x + 19$

$\frac{1}{2}x + 2x = 19 - 5\frac{1}{2}$

$2\frac{1}{2}x = 13\frac{1}{2}$

$x = 5.4$

$y = -2(5.4) + 19$

$y = 8.2$

The coordinates of *P* are (5.4, 8.2).

Height of triangle *JDL*: $PL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8 - 5.4)^2 + (3 - 8.2)^2}$

$= \sqrt{(2.6)^2 + (-5.2)^2}$

$= \sqrt{33.8}$

≈ 5.8

Area of triangle *JDL*: $A = \frac{1}{2}bh$

$= \frac{1}{2}(JD)(PL)$

$= \frac{1}{2}(3\sqrt{5})(\sqrt{33.8})$

$= \frac{1}{2}(3\sqrt{169})$

$= 19.5$

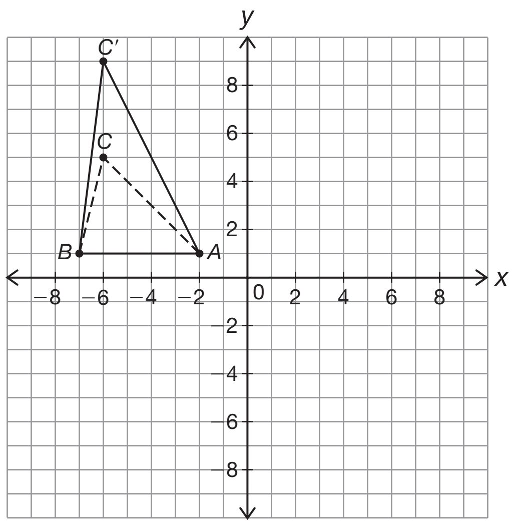
The area of triangle *JDL* is 19.5 square units.

14.2 Doubling the Area of a Triangle

To double the area of a triangle, only the length of the base or the height of the triangle need to be doubled. If both the length of the base and the height are doubled, the area will quadruple.

Example

Double the area of triangle *ABC* by manipulating the height.



Area of *ABC*

$A = \frac{1}{2}bh$

$= \frac{1}{2}(5)(4)$

$= 10$

Area of *ABC′*

$A = \frac{1}{2}bh$

$= \frac{1}{2}(5)(8)$

$= 20$

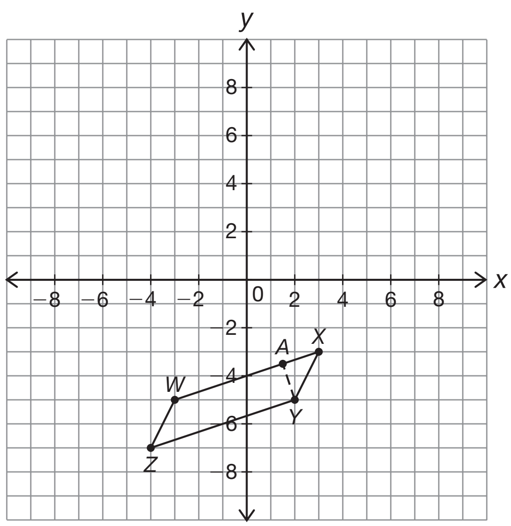
By doubling the height, the area of triangle *ABC′* is double the area of triangle *ABC*.

14.3 Determining the Perimeter and Area of Parallelograms on the Coordinate Plane

The formula for calculating the area of a parallelogram is the same as the formula for calculating the area of a rectangle: $A = bh$. The height of a parallelogram is the length of a perpendicular line segment from the base to a vertex opposite the base.

Example

Determine the perimeter and area of parallelogram *WXYZ*.



The vertices of parallelogram *WXYZ* are $W(-3, -5)$, $X(3, -3)$, $Y(2, -5)$, and $Z(-4, -7)$.

$$\begin{aligned} WX &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - (-3))^2 + (-3 - (-5))^2} \\ &= \sqrt{6^2 + 2^2} \\ &= \sqrt{40} \\ &= 2\sqrt{10} \end{aligned}$$

$$\begin{aligned} YZ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-4 - 2)^2 + (-7 - (-5))^2} \\ &= \sqrt{(-6)^2 + (-2)^2} \\ &= \sqrt{40} \\ &= 2\sqrt{10} \end{aligned}$$

$$\begin{aligned} WZ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-4 - (-3))^2 + (-7 - (-5))^2} \\ &= \sqrt{(-1)^2 + (-2)^2} \\ &= \sqrt{5} \end{aligned}$$

$$\begin{aligned} XY &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2 - 3)^2 + (-5 - (-3))^2} \\ &= \sqrt{(-1)^2 + (-2)^2} \\ &= \sqrt{5} \end{aligned}$$

$$\begin{aligned} P &= WX + XY + YZ + WZ \\ &= 2\sqrt{10} + \sqrt{5} + 2\sqrt{10} + \sqrt{5} \\ &\approx 17.1 \end{aligned}$$

The perimeter of parallelogram *WXYZ* is approximately 17.1 units.

To determine the area of parallelogram *WXYZ*, first calculate the height, *AY*.

$$\text{Slope of base } WX\text{: } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-5)}{3 - (-3)} = \frac{2}{6} = \frac{1}{3}$$

Slope of height *AY*: $m = -3$

Equation of base *WX*: $(y - y_1) = m(x - x_1)$

$$\begin{aligned} (y - (-3)) &= \frac{1}{3}(x - 3) \\ y &= \frac{1}{3}x - 4 \end{aligned}$$

Equation of height *AY*: $(y - y_1) = m(x - x_1)$

$$\begin{aligned} (y - (-5)) &= -3(x - 2) \\ y &= -3x + 1 \end{aligned}$$

Intersection of *WX* and *AY*, or *A*: $\frac{1}{3}x - 4 = -3x + 1$

$$\begin{aligned} \frac{1}{3}x + 3x &= 1 + 4 \\ \frac{10}{3}x &= 5 \\ x &= 1\frac{1}{2} \end{aligned}$$

$$\begin{aligned} y &= -3x + 1 \\ y &= -3\left(1\frac{1}{2}\right) + 1 \\ y &= -3\frac{1}{2} \end{aligned}$$

The coordinates of point A are $\left(1\frac{1}{2}, -3\frac{1}{2}\right)$.

$$AY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(2 - 1\frac{1}{2}\right)^2 + \left(-5 - \left(-3\frac{1}{2}\right)\right)^2}$$

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(-1\frac{1}{2}\right)^2}$$

$$= \sqrt{2.5}$$

Area of parallelogram $WXYZ$: $A = bh$

$$A = 2\sqrt{10}(\sqrt{2.5})$$

$$A = 10$$

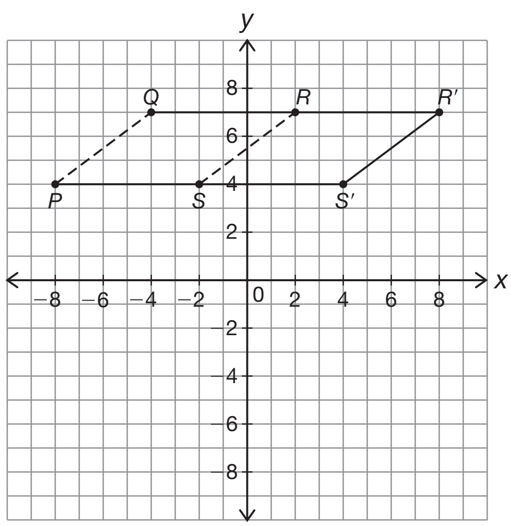
The area of parallelogram $WXYZ$ is 10 square units.

14.3 Doubling the Area of a Parallelogram

To double the area of a parallelogram, only the length of the bases or the height of the parallelogram needs to be doubled. If both the length of the bases and the height are doubled, the area will quadruple.

Example

Double the area of parallelogram $PQRS$ by manipulating the length of the bases.



Area of $PQRS$	Area of $PQR'S'$
$A = bh$	$A = bh$
$= (6)(3)$	$= (12)(3)$
$= 18$	$= 36$

By doubling the length of the bases, the area of parallelogram $PQR'S'$ is double the area of parallelogram $PQRS$.

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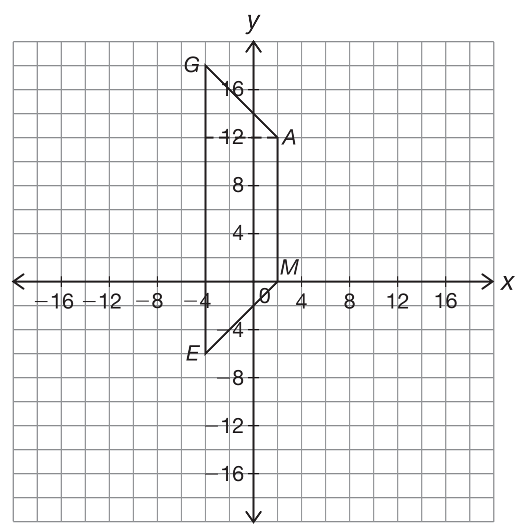
14.4 Determining the Perimeter and Area of Trapezoids on the Coordinate Plane

A trapezoid is a quadrilateral that has exactly one pair of parallel sides. The parallel sides are known as the bases of the trapezoid, and the non-parallel sides are called the legs of the trapezoid. The area of a trapezoid can be calculated by using the formula $A = \left(\frac{b_1 + b_2}{2}\right)h$, where b_1 and b_2 are the bases of the trapezoid and h is a perpendicular segment that connects the two bases.

Example

Determine the perimeter and area of trapezoid *GAME*.



The coordinates of the vertices of trapezoid *GAME* are $G(-4, 18)$, $A(2, 12)$, $M(2, 0)$, and $E(-4, -6)$.

$$GA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2 - (-4))^2 + (12 - 18)^2}$$
$$= \sqrt{6^2 + (-6)^2}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2}$$

$$ME = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{((-4) - 2)^2 + ((-6) - 0)^2}$$
$$= \sqrt{(-6)^2 + (-6)^2}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2}$$

$$EG = 18 - (-6)$$
$$= 24$$

$$AM = 12 - 0$$
$$= 12$$

$$P = GA + AM + ME + EG$$
$$= 6\sqrt{2} + 12 + 6\sqrt{2} + 24$$
$$\approx 53.0$$

The perimeter of trapezoid *GAME* is approximately 53.0 units.

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The height of trapezoid *GAME* is 6 units.

$$A = \left(\frac{b_1 + b_2}{2}\right)h$$
$$= \left(\frac{24 + 12}{2}\right)(6)$$
$$= 108$$

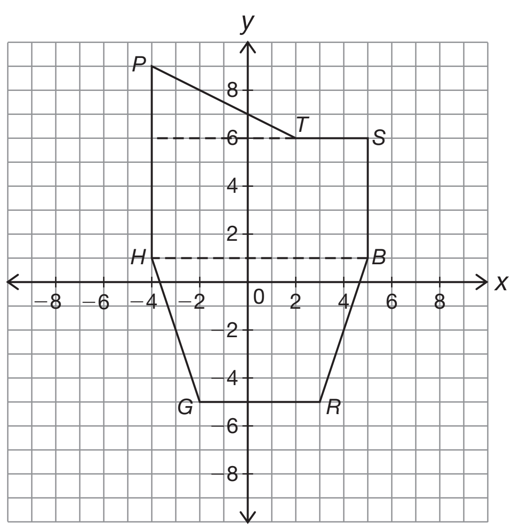
The area of trapezoid *GAME* is 108 square units.

14.4 Determining the Perimeter and Area of Composite Figures on the Coordinate Plane

A composite figure is a figure that is formed by combining different shapes. The area of a composite figure can be calculated by drawing line segments on the figure to divide it into familiar shapes and determining the total area of those shapes.

Example

Determine the perimeter and area of the composite figure.



The coordinates of the vertices of this composite figure are $P(-4, 9)$, $T(2, 6)$, $S(5, 6)$, $B(5, 1)$, $R(3, -5)$, $G(-2, -5)$, and $H(-4, 1)$.

$TS = 3$, $SB = 5$, $RG = 5$, $HP = 8$

$$PT = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - (-4))^2 + (6 - 9)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{45} = 3\sqrt{5}$$

$$BR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3 - 5)^2 + (-5 - 1)^2} = \sqrt{(-2)^2 + (-6)^2} = \sqrt{40} = 2\sqrt{10}$$

$$GH = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-4 - (-2))^2 + (1 - (-5))^2} = \sqrt{(-2)^2 + (6)^2} = \sqrt{40} = 2\sqrt{10}$$

$$P = PT + TS + SB + BR + RG + GH + HP$$
$$= 3\sqrt{5} + 3 + 5 + 2\sqrt{10} + 5 + 2\sqrt{10} + 8$$
$$\approx 40.4$$

The perimeter of the composite figure *PTSBRGH* is approximately 40.4 units.

The area of the figure is the sum of the triangle, rectangle, and trapezoid formed by the dotted lines.

Area of triangle:

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(6)(3)$$
$$= 9$$

Area of rectangle:

$$A = bh$$
$$= 9(5)$$
$$= 45$$

Area of trapezoid:

$$A = \left(\frac{b_1 + b_2}{2}\right)h$$
$$= \left(\frac{9 + 5}{2}\right)(6)$$
$$= 42$$

The area of composite figure: $A = 9 + 45 + 42$
$= 96$

The area of the composite figure *PTSBRGH* is 96 square units.