Connecting Algebra and Geometry with Polygons

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15.1

Name That Triangle!

Classifying Triangles on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

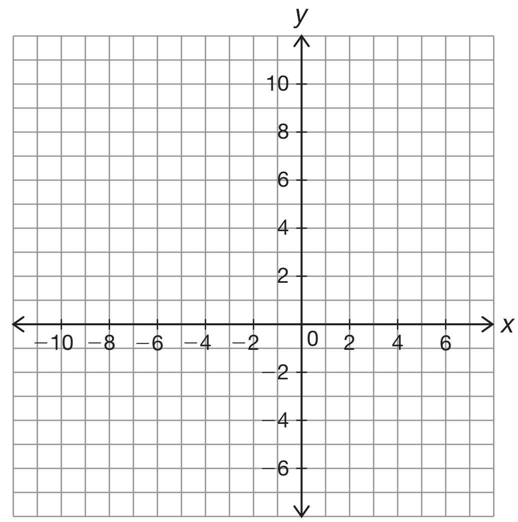
- Determine the coordinates of a third vertex of a triangle, given the coordinates of two vertices and a description of the triangle.
- Classify a triangle given the locations of its vertices on a coordinate plane.

Since you may soon be behind the steering wheel of a car, it is important to know the meaning of the many signs you will come across on the road. One of the most basic is the yield sign. This sign indicates that a driver must prepare to stop to give a driver on an adjacent road the right of way. The first yield sign was installed in the U.S. in 1950 in Tulsa, Oklahoma, and had been designed by a police officer of the town. Originally the sign was shaped like a keystone, but over time it was changed. Today it is an equilateral triangle and is used just about everywhere in the world. While some countries may use different colors or wording (some countries call it a "give way" sign) the signs are all the same in size and shape.

Why do you think road signs tend to be different, but basic shapes, such as rectangles, triangles, and circles? Would it matter if a stop sign was an irregular heptagon? Does the shape of a sign make it any easier or harder to recognize?

PROBLEM 1 Location, Location, Location!

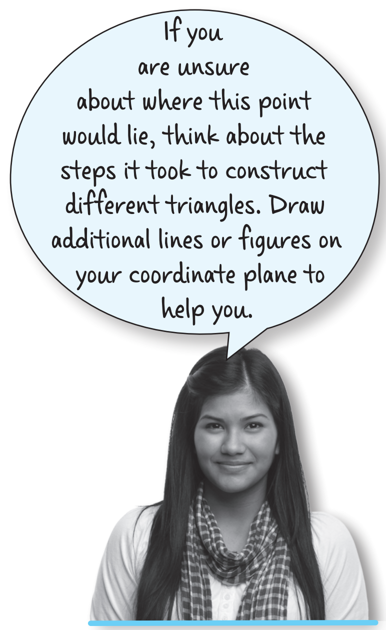
1. Graph line segment AB using points $A(-6, 7)$ and $B(-6, 3)$.



2. Recall that triangles can be classified by the measures of their angles as acute, right, or obtuse. Using line segment AB as one side of a triangle, determine the location for point C such that triangle ABC is:

   a. a right triangle.

   b. an acute triangle.



c. an obtuse triangle.

3. Recall that triangles can also be classified by the measures of their sides as equilateral, isosceles, and scalene. Using line segment *AB* as one side of a triangle, determine the location for point *C* such that triangle *ABC* is:

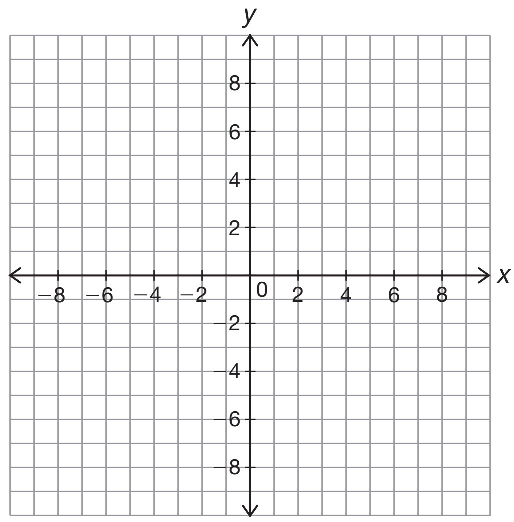
a. an equilateral triangle.

b. an isosceles triangle.

c. a scalene triangle.

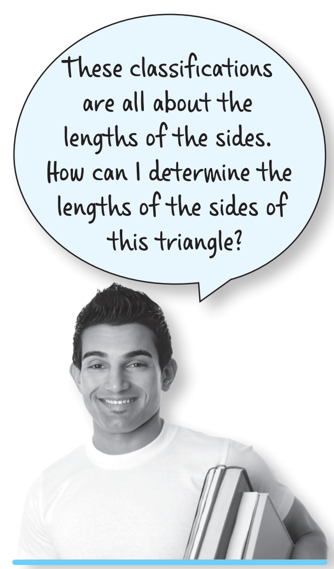
PROBLEM 2 What's Your Name Again?

1. Graph triangle *ABC* using points *A*(0, −4), *B*(0, −9), and *C*(−2, −5).



2. Classify triangle *ABC*.

   a. Determine if triangle *ABC* is scalene, isosceles, or equilateral. Explain your reasoning.



b. Determine if triangle *ABC* is a right triangle. Explain your reasoning. If it is not a right triangle, use a protractor to determine what type of triangle it is.

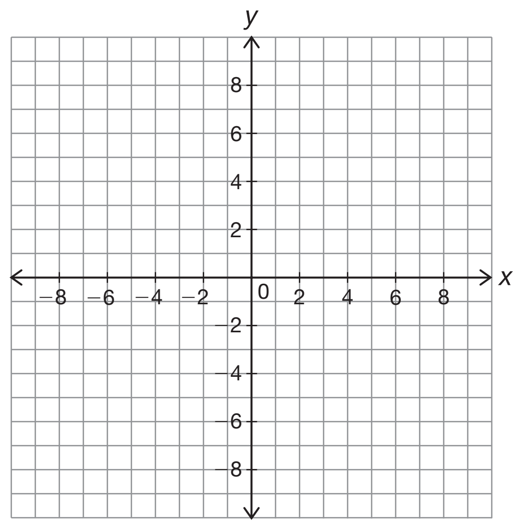
c. Zach does not like using the slope formula. Instead, he decides to use the Pythagorean Theorem to determine if triangle *ABC* is a right triangle since he already determined the lengths of the sides. His work is shown.

Zach

$$a^2 - b^2 = c^2$$
$$(\sqrt{5})^2 + (\sqrt{20})^2 = 5^2$$
$$5 + 20 = 25$$
$$25 = 25$$

He determines that triangle *ABC* must be a right triangle because the sides satisfy the Pythagorean Theorem. Is Zach's reasoning correct? Explain why or why not.

3. Graph triangle *ABC* using points *A*(−2, 4), *B*(8, 4), and *C*(6, −2).



4. Classify triangle *ABC*.

   a. Determine if triangle *ABC* is a scalene, isosceles, or equilateral triangle. Explain your reasoning.

b. Determine if triangle *ABC* is a right triangle. Explain your reasoning. If it is not a right triangle, use a protractor to determine what type of triangle it is.

Be prepared to share your solutions and methods.

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Name That Quadrilateral!

15.2

Classifying Quadrilaterals on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

- Determine the coordinates of a fourth vertex, given the coordinates of three vertices of a quadrilateral and a description of the quadrilateral.
- Classify a quadrilateral given the locations of its vertices on a coordinate plane.

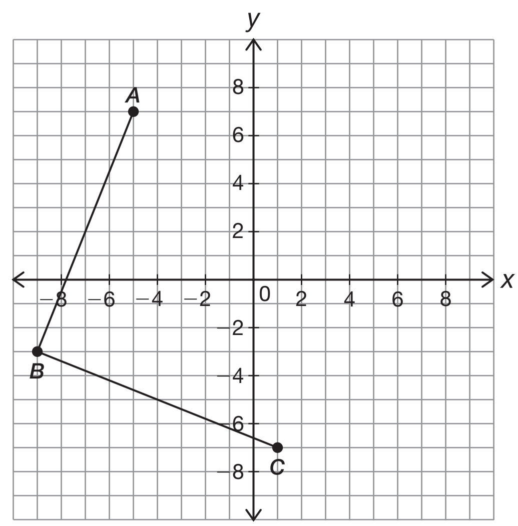
Have you ever mistaken a stranger for someone you know? While it may be embarrassing, this type of mistake is usually no big deal. However, some people struggle with a disorder known as facial blindness where their ability to recognize people's faces is impaired. Studies have found that this disorder usually occurs after a head trauma but it may also be genetic. Some people with this disorder can see the different parts of people faces—forehead, cheekbones, nose—however, their brains cannot put the pieces together to make the face. These people struggle to recognize their friends, parents, and even their children! While there is no cure for this disorder, many people are able to compensate by identifying a key feature of a person such as a haircut or moustache.

Think about a family member or good friend. Can you describe them? What type of features help you recognize this person from others who may look similar? Do you use some of the same features to describe other creatures or objects?

PROBLEM 1 Where Can It Go?

Recall that quadrilaterals can be described by the lengths of their sides and by the measures of their angles. The four quadrilaterals you will use in this lesson are parallelograms, squares, rectangles, and trapezoids.

1. Analyze the given points *A*, *B*, and *C*. You want to plot point *D* such that quadrilateral *ABCD* is a square.



2. Even though point *D* is not in place, the fact that *ABCD* will be a square gives you the information needed to identify the location of point *D*.

   a. How does knowing that a square has two pairs of parallel sides help when determining the unknown point?

   b. How does knowing that a square has 4 right angles help when determining the unknown point?

3. Determine the location of point D. Plot and label point D on the coordinate plane.

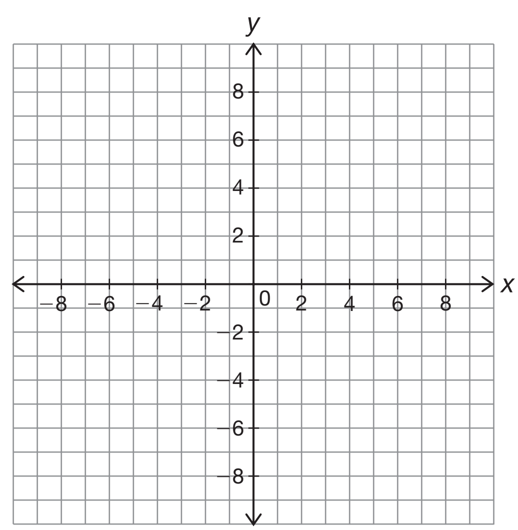
4. Using the same locations for points A, B, and C, identify the location of point D, such that quadrilateral $ABCD$ is a trapezoid.

a. Identify information about a trapezoid that is helpful in determining the location of point D. Explain your reasoning.

b. Describe the possible locations of point D such that quadrilateral $ABCD$ is a trapezoid.

PROBLEM 2 Which One Are You Again?

1. Graph quadrilateral $ABCD$ using points $A(-5, 6)$, $B(-8, 2)$, $C(-5, -2)$, and $D(-2, 2)$.



2. Determine if quadrilateral *ABCD* can best be described as a trapezoid, a rhombus, a rectangle, a square, or none of these.

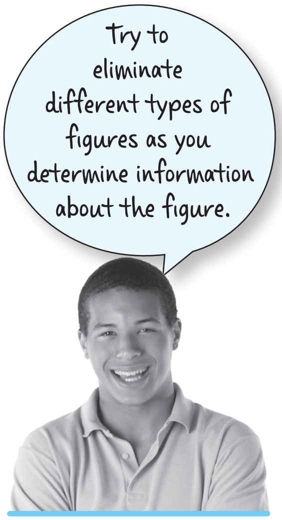
a. Determine the length of each side of quadrilateral *ABCD*.

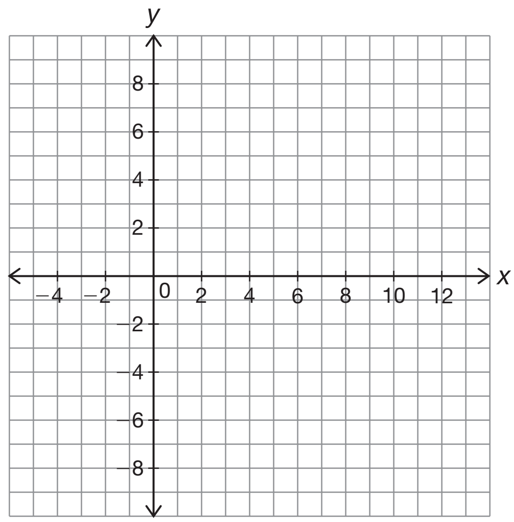
b. Can you classify quadrilateral *ABCD* from its side lengths? If so, identify the type of figure. If not, explain why not.

c. Determine the slope of each line segment in the quadrilateral.

d. Describe the relationship between the slopes. Can you now identify the figure? If so, identify the type of figure. If not, explain why not.

3. Graph quadrilateral *ABCD* using points *A*(6, 2), *B*(4, −4), *C*(10, −6), and *D*(12, 0). Determine if this quadrilateral can best be described as a trapezoid, a rhombus, a rectangle, a square, or none of these. Explain your reasoning.





Be prepared to share your solutions and methods.

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15.3

Is That Point on the Circle?

Determining Points on a Circle

LEARNING GOALS

In this lesson, you will:

- Determine if a point lies on a circle on the coordinate plane given the circle's center at the origin, the radius of the circle, and the coordinates of the point.
- Determine if a point lies on a circle on the coordinate plane given the circle's center not at the origin, the radius of the circle, and the coordinates of the point.
- Transform a circle about the coordinate plane and determine if a point lies on a circle's image given the pre-image's center, radius, and the coordinates of the point.

In the United States, most of the borders separating our states are straight lines or slightly irregular. This is because the borders were often decided by lines of longitude and latitude, meandering rivers, hulking mountain ranges, or other natural formations. However, there is an area on the Pennsylvania-Delaware border that is an arc. This arc, known as the Twelve-Mile Circle, has a radius of exactly 12 miles and is centered on the city of New Castle, Delaware's courthouse. This odd boundary was formed in 1682 when the Duke of York wrote out the original deed of Delaware. In this deed, the Duke claimed that all land within this twelve-mile circle that was not already a part of New Jersey or Maryland, including the Delaware River, would be a part of Delaware. Go take a look at a map of the United States and you will find the Twelve-Mile Circle separating Pennsylvania from Delaware.

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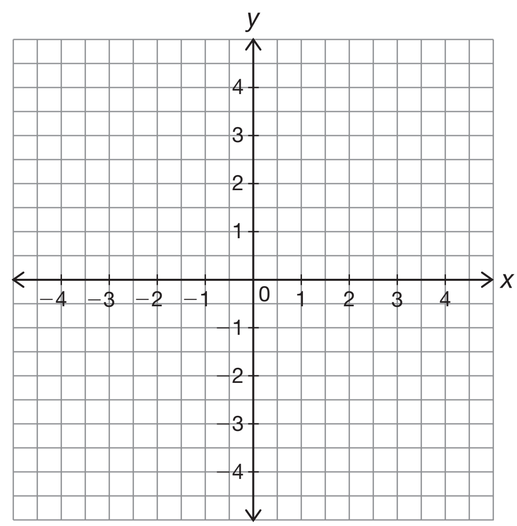
PROBLEM 1 The Origin Is the Center

In this lesson, you will explore the connection between the Pythagorean Theorem and a circle which has a center at the origin.

1. Graph circle A so that the circle's center is at the origin, and contains a point P at (0, 5).



2. Analyze the circle you graphed in Question 1.

   a. Graph line segment AP. What is this line segment called in reference to circle A?

   b. A point, Q, is plotted on the y-axis so it sits on circle A. Describe the relationship between the length of line segment AQ and line segment AP. Explain your reasoning.

   c. How can you determine if circle A passes through a specific point that is not on either the x- or y-axis?

3. Plot point *B* on the coordinate plane at (4, 3). Do you think point *B* is on circle *A*? Explain your reasoning.

Points that do not lie exactly on gridlines may appear to be at certain locations, but you must use algebra to show their exact locations.

4. Analyze circle *A* and point *B* that you graphed.

a. What information can you use from the graph to determine if point *B* is on circle *A*? Explain why you can use this information.

b. Determine the length of segment *AB* using the Distance Formula. Show your work.

c. Determine the length of segment *AB* using the Pythagorean Theorem. First, connect points *A* and *B* together to create the hypotenuse of a right triangle. Then, draw a perpendicular line segment from point *B* to the *x*-axis. Then use the dimensions to determine the length of segment *AB*.

d. What do you notice about the solutions using the Distance Formula and the Pythagorean Theorem?

5. What do you notice about the length of *AB* and the radius of circle *A*? How might you use this information to determine if point *B* is on circle *A*?

6. Does point *B* lie on circle *A*? Explain your reasoning.

7. Circle *D* is centered at the origin and has a diameter of 16 units. Determine if point *H*, located at $(5, \sqrt{38})$, lies on circle *D*.

a. What is the radius of circle *D*? How do you know?

b. Do you know the exact value of $\sqrt{38}$? Explain why or why not.

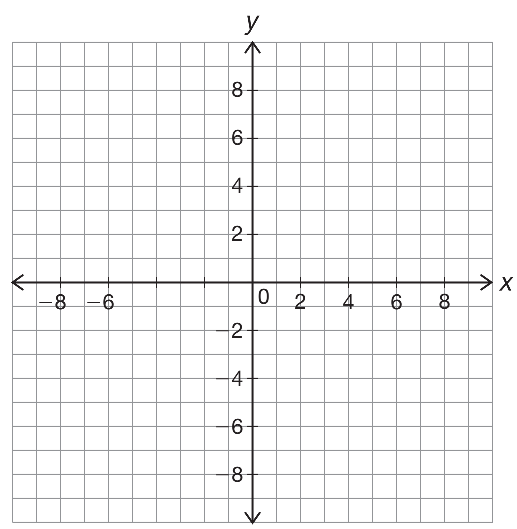
c. Describe which method you will use to determine if point *H* lies on circle *D*. Explain why you will use this method.

d. Does point *H* lie on circle *D*? Show your work and explain how you determined your answer.

PROBLEM 2 Oh No! The Center *Isn't* at the Origin?

A figure can be in any of the four quadrants of a coordinate plane—and sometimes they are in multiple quadrants. This is also true of circles.

1. Circle *K* has its center at (−2, −3), and contains point *N*(−4, −1.5). Determine if point *C* at (−2, −5.5) lies on circle *K*.



a. Describe what method(s) you will use to determine whether point C lies on circle K.

b. Determine whether point C lies on circle K. Describe how you determined your answer.

2. Suppose circle K is reflected over the x-axis. Determine if point C' at (0.5, 5.25) lies on the image circle K'.

a. Determine the center of circle K'. How can you determine the image's center without graphing?

b. What is the radius of circle K'? How do you know?

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c. Describe how you can determine if point C' lies on circle K'. Then determine if point C' does lie on circle K'.

Be prepared to share your solutions and methods.

15

Name That Point on the Circle

Circles and Points on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

- Determine the coordinates of a point that lies on a circle given the location of the center point and the radius of the circle.
- Use the Pythagorean Theorem to determine the coordinates of a point.

While on the road you may have seen someone off to the side with a tripod and what looked like a camera. Unless you were at a scenic location, chances are that person wasn't taking a picture. This person may be a surveyor and if so he or she is taking measurements that are often used to make maps and determine boundaries. Surveyors must use their geometry and trigonometry skills regularly in order to determine the three-dimensional location of points and the distances and angles between them. Until very recently, the tools of a surveyor were relatively basic: a tape measure to measure distance, a level to measure height differences, and a theodolite, which is the instrument that sits on top of the tripod and measures angles. Today, surveyors are able to use GPS systems to gather most of these measurements much more quickly; however, the measurements are not always as accurate as measuring by hand, and as many people know, GPS is difficult to use in a densely wooded area or if there is a lot of cloud cover.

While working through this lesson keep the job of a surveyor in mind. While you will be working on a flat coordinate plane, the surveyor often has obstacles and obstructions in his or her way. How might the surveyor get around these obstacles when making measurements? How might these affect the accuracy of the measurements?

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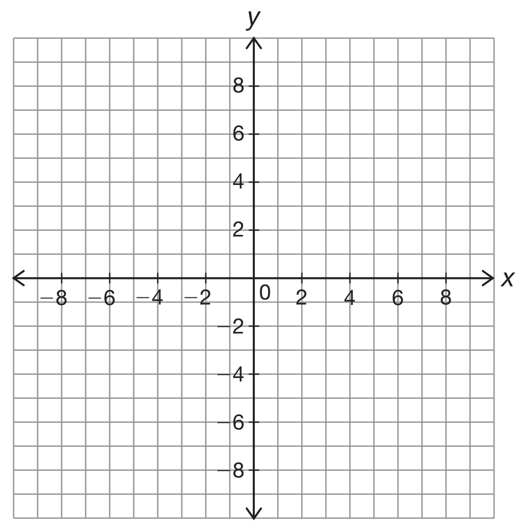
PROBLEM 1 The Origin Is My Center

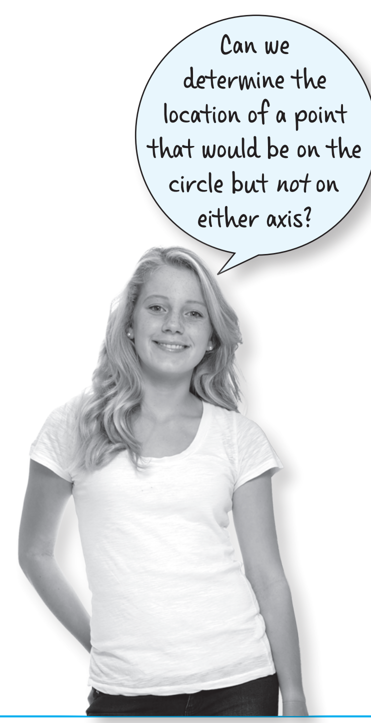
In the previous lesson, you determined whether a given point was located on a circle. You worked with circles that were centered either on the origin or not. Now, given a circle, you will determine the locations of multiple points on the circle.

1. Circle A is centered at the origin and has a diameter of 20 units.

a. Graph circle A.

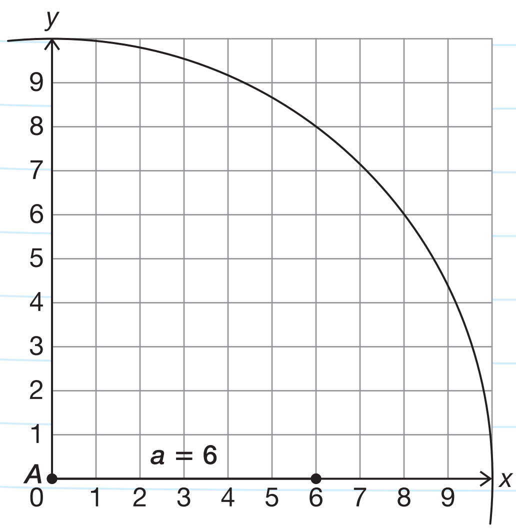


b. Determine the coordinates of four points on the circle such that two points lie on the x-axis and two points lie on the y-axis. Explain how you determined your answers.

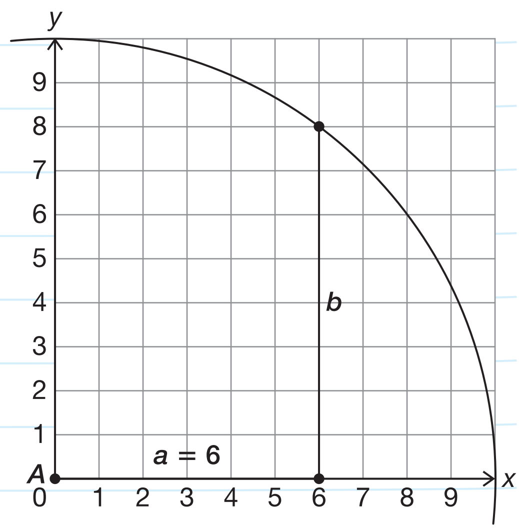


To determine a point in a quadrant that lies on a circle, but not on an axis, follow the steps shown.

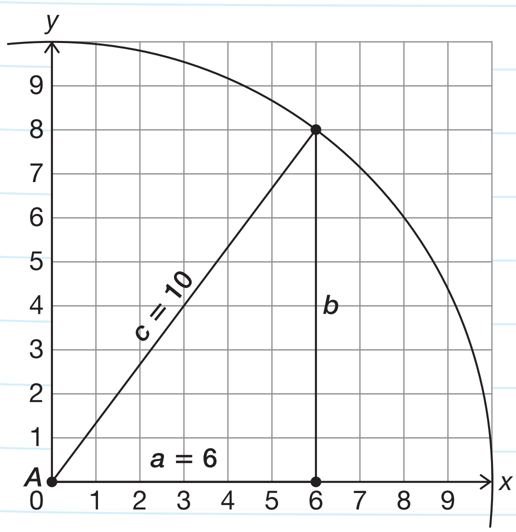
Step 1: Construct a line from the center to a point on the *x*-axis less than the radius. This will represent one side of a right triangle.



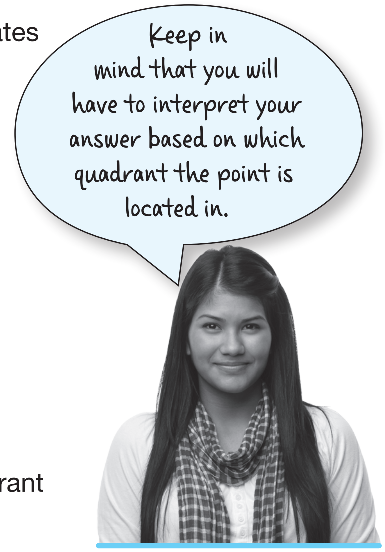
Step 2: Draw a vertical line connecting the end of this side to the circle. This represents the second side of a right triangle.



Step 3: Connect the intersection point of the vertical side of your triangle and the center of the circle. This represents the hypotenuse.

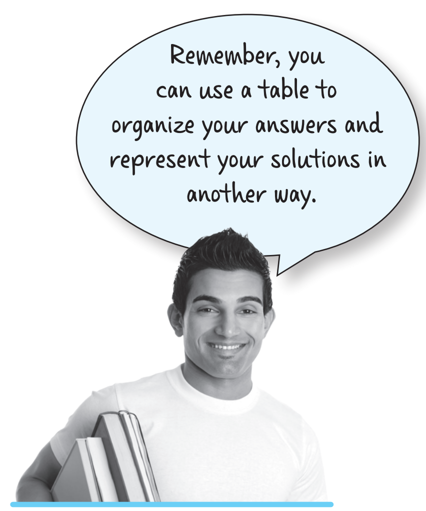


2. Use the Pythagorean Theorem to determine the coordinates of the point that lies on circle A in Quadrant I.



3. Determine the coordinates of a point in the second quadrant that lies on circle A. Use algebra to support your answer.

4. Determine the coordinates of a point in the third quadrant that lies on circle A. Use algebra to support your answer.



5. Determine the coordinates of a point in the fourth quadrant that lies on circle A. Use algebra to support your answer.

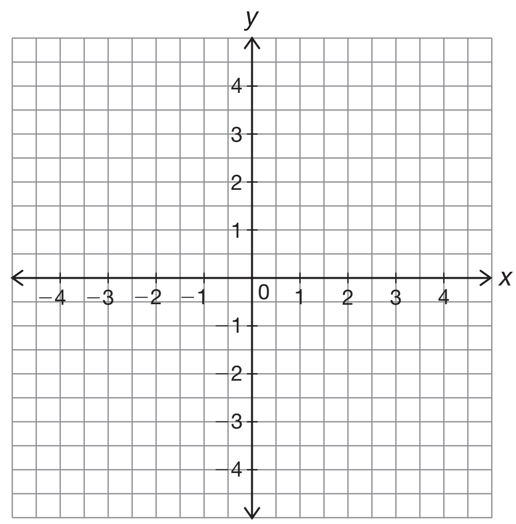
6. Use your answers from Questions 2 through 5 to complete the table.

Radius of circle *A*	*x*-intercepts	*y*-intercepts	Point in Quad I	Point in Quad II	Point in Quad III	Point in Quad IV

Determining points that lie on a circle can seem pretty simple when the points have whole number coordinates. However, more challenges arise when you are determining the coordinates of a point and only one of the coordinates is a whole number.

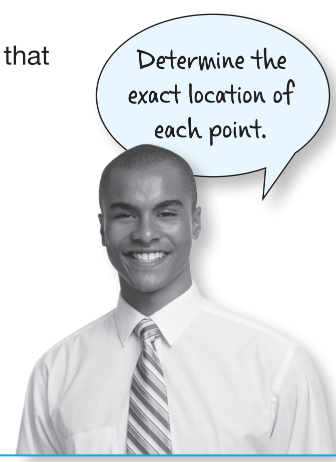
7. Circle *B* is centered at the origin and has a diameter of 8 units.

a. Graph circle *B*.



b. Determine the coordinates of four points on the circle such that two points lie on the *x*-axis and two points lie on the *y*-axis.

c. Determine the coordinates of a point in the first quadrant that lies on circle *B*. Use algebra to support your answer.



d. Determine the coordinates of a point in the second quadrant that lies on circle *B*. Use algebra to support your answer.

e. Determine the coordinates of a point in the third quadrant that lies on circle *B*. Use algebra to support your answer.

f. Determine the coordinates of a point in the fourth quadrant that lies on circle *B*. Use algebra to support your answer.

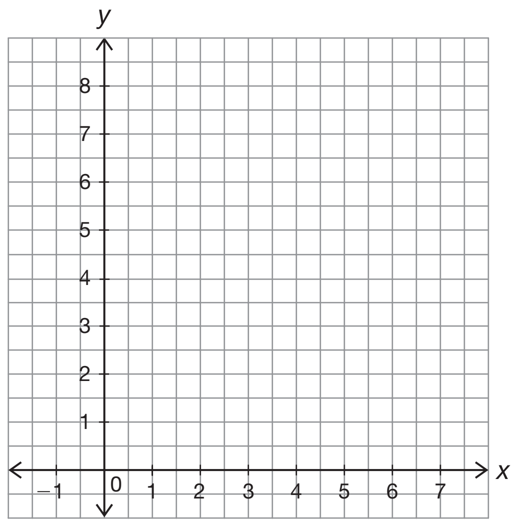
g. Use your answers from parts (c) through (f) to complete the table.

Radius of circle *B*	*x*-Intercepts	*y*-Intercepts	Point in Quad I	Point in Quad II	Point in Quad III	Point in Quad IV

PROBLEM 2 The Circle Is Movin' Up!

1. Circle C has a center at point (2, 5) with a radius of 3 units.

   a. Graph circle C.



   b. Determine the coordinates of the points on the circle that lie directly above and directly below the center point of the circle. Explain how you determined these coordinates.

   c. Determine the coordinates of the points on the circle that lie directly to the left and directly to the right of the center point of the circle. Explain how you determined these coordinates.

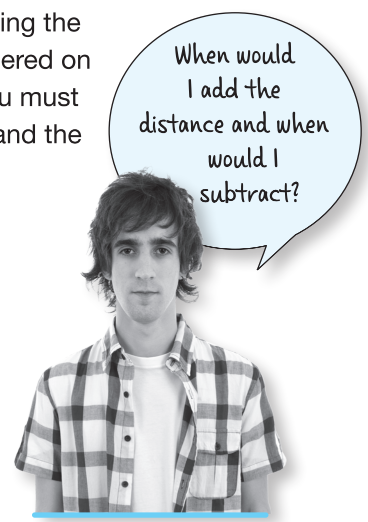
2. Carlos determined the coordinates of an additional point that lies on circle C. His work is shown.

$$a^2 + b^2 = c^2$$
$$1^2 + b^2 = 3^2$$
$$1 + b^2 = 9$$
$$b^2 = 8$$
$$b = \sqrt{8}$$

Point $(1, \sqrt{8})$ lies on circle C.

Determine why Carlos' solution is incorrect. Then determine the correct coordinates of the point and plot it on the coordinate plane in Question 1.

Carlos's calculations bring up a new challenge. When determining the exact coordinates of a point that lies on a circle that is not centered on the origin, an additional step is required. In these situations, you must either add or subtract the vertical distance between the origin and the circle's center to the *y*-coordinate.



3. Use transformations to determine the coordinates of a different point that lies on circle C.

a. Explain how you can use reflection to locate a different point that lies on circle C.

b. Use reflection to determine the coordinates of a different point that lies on circle C.

4. Analyze the points of circle C.

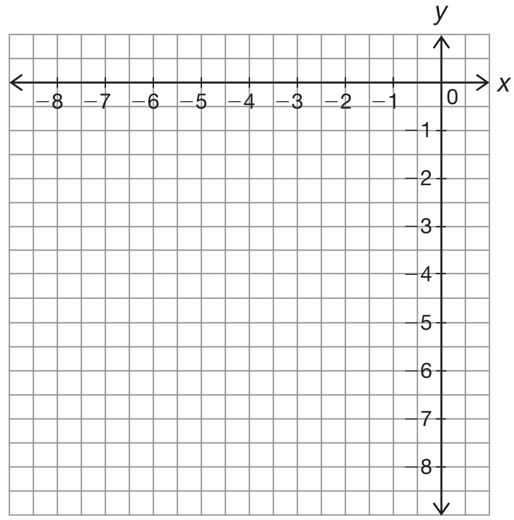
a. Complete the table by using your answers from Questions 1 through 3.

Center	Radius	Points Above and Below Center	Points to the Left and Right of Center	Point on circle C (not on x- or y-axis)	Additional point on circle C (result of reflection)

b. What patterns do you notice in the table?

5. Circle D is centered at point $(-6, -5)$ with a radius of 2 units.

   a. Graph circle D.



   b. Complete the table using circle D, that you graphed in Question 5 part (a).

Center	Radius	Points Above and Below the Center	Points to the Left and Right of Center	Point on circle *D* (not on *x*- or *y*-axis)	Additional point on circle *D* (result of transformation)

Be prepared to share your solutions and methods.

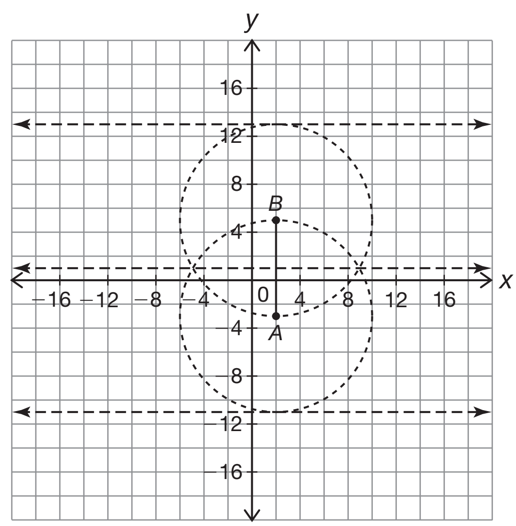
Chapter 15 Summary

15.1 Determining the Third Vertex of a Triangle Given Two Points

A line segment formed by two points on the coordinate plane can represent one side of a triangle. The placement of the third point depends on the type of triangle being formed.

Example

The line segment *AB* has been graphed on the coordinate plane for the points *A*(2, −3) and *B*(2, 5). Determine the location of point *C* such that triangle *ABC* is an acute triangle.



To create an acute triangle, point *C* can have an infinite number of locations as long as the location satisfies one of the following conditions:

- Point *C* could be located anywhere on circle *A* between the *y*-values of −3 and 5.
- Point *C* could be located anywhere on circle *B* between the *y*-values of −3 and 5.

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15.1 Describing a Triangle Given Three Points on a Coordinate Plane

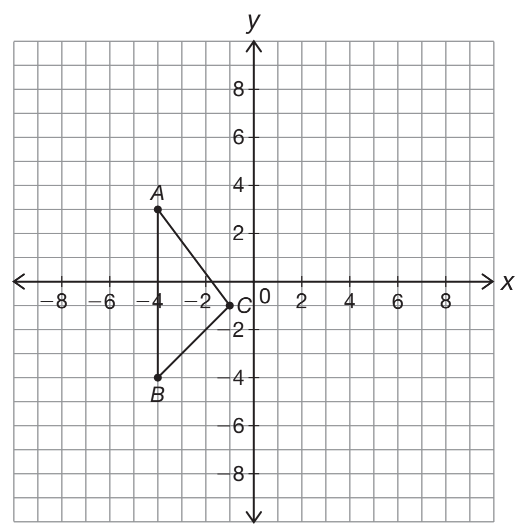
When given three points on the coordinate plane, the triangle formed can be described by the measures of its sides and angles. To determine if the triangle is scalene, isosceles, or equilateral, use the Distance Formula to determine the length of each side. To determine if the triangle is right, use the slope formula or the Pythagorean Theorem. If the triangle is not right, a protractor can be used to determine if it is obtuse or acute.

Example

Describe triangle *ABC* with points $A(-4, 3)$, $B(-4, -4)$, and $C(-1, -1)$.

$AB = 3 - (-4) = 7$

$$
\begin{aligned}
BC &= \sqrt{(-1-(-4))^2 + (-1-(-4))^2} \\
&= \sqrt{3^2 + 3^2} \\
&= \sqrt{9 + 9} \\
&= \sqrt{18}
\end{aligned}
$$



$$
\begin{aligned}
AC &= \sqrt{(-1-(-4))^2 + (-1-3)^2} \\
&= \sqrt{3^2 + (-4)^2} \\
&= \sqrt{9 + 16} \\
&= \sqrt{25} \\
&= 5
\end{aligned}
$$

Each side is a different length, so triangle *ABC* is scalene.

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
(\sqrt{18})^2 + 5^2 &= 7^2 \\
18 + 25 &= 49 \\
43 &\neq 49
\end{aligned}
$$

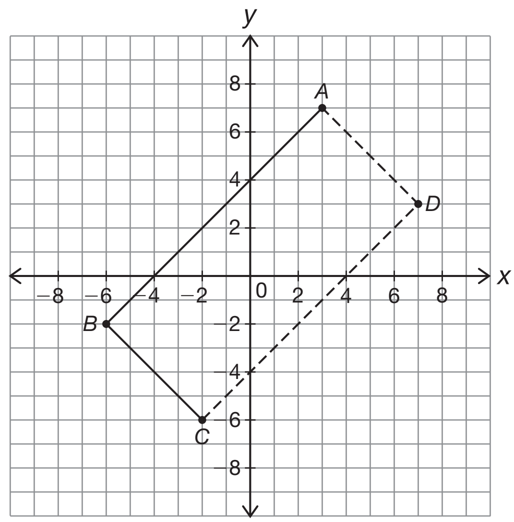
The side lengths do not satisfy the Pythagorean Theorem, so triangle *ABC* is not a right triangle. The triangle is acute because all of the angles are less than 90°.

15.2 Determining the Fourth Vertex of a Quadrilateral Given Three Points

When given three points on a coordinate plane, the location of the fourth point can affect the type of quadrilateral created. Slope and point-slope formulas can be used to determine the location of a fourth point.

Example

The points $A(3, 7)$, $B(-6, -2)$, and $C(-2, -6)$ are plotted. Determine the location for point D such that quadrilateral $ABCD$ is a rectangle.



Slope of line segments AB and CD:

$$
\begin{aligned}
m &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{-2 - 7}{-6 - 3} \\
&= \frac{-9}{-9} \\
&= 1
\end{aligned}
$$

The slope of BC and AD must be the negative reciprocal of the slope of AB and CD. The slope of BC and AD is -1.

Equation of a line with a slope of -1 passing through point A:

$$
\begin{aligned}
(y - y_1) &= m(x - x_1) \\
(y - 7) &= -1(x - 3) \\
y - 7 &= -x + 3 \\
y &= -x + 10
\end{aligned}
$$

Equation of a line with a slope of 1 passing through point C:

$$
\begin{aligned}
(y - y_1) &= m(x - x_1) \\
(y - (-6)) &= 1(x - (-2)) \\
y + 6 &= x + 2 \\
y &= x - 4
\end{aligned}
$$

Solution to the system of equations:

$$
\begin{aligned}
-x + 10 &= x - 4 \qquad & y &= x - 4 \\
-2x &= -14 & &= 7 - 4 \\
x &= 7 & &= 3
\end{aligned}
$$

The coordinates of point D are $(7, 3)$.

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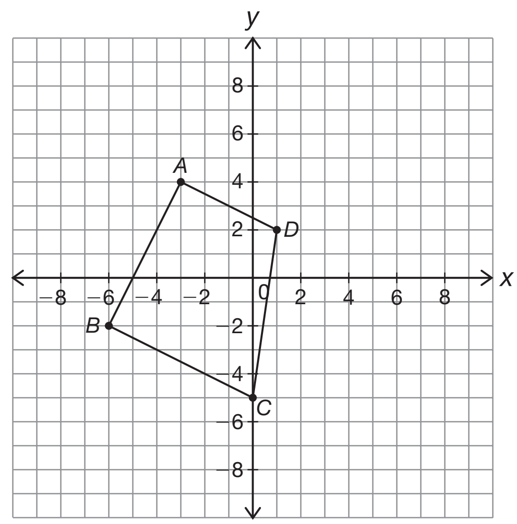
15.2 Describing a Quadrilateral Given Four Points on a Coordinate Plane

A quadrilateral on the coordinate plane can be classified as a parallelogram, trapezoid, square, or rectangle. To identify the quadrilateral, use the distance formula to determine the length of each side. Then use the slope formula to determine the angles. If the slopes of two line segments are negative reciprocals, the line segments are perpendicular, and form a right angle. If the slopes of two line segments are equal, the line segments are parallel.

Example

Describe quadrilateral *ABCD*.



$$AB = \sqrt{(-6-(-3))^2 + (-2-4)^2}$$
$$= \sqrt{(-3)^2 + (-6)^2}$$
$$= \sqrt{9 + 36}$$
$$= \sqrt{45}$$

$$BC = \sqrt{(0-(-6))^2 + (-5-(-2))^2}$$
$$= \sqrt{(6)^2 + (-3)^2}$$
$$= \sqrt{36 + 9}$$
$$= \sqrt{45}$$

$$CD = \sqrt{(1-0)^2 + (2-(-5))^2}$$
$$= \sqrt{(1)^2 + (7)^2}$$
$$= \sqrt{1 + 49}$$
$$= \sqrt{50}$$

$$AD = \sqrt{(1-(-3))^2 + (2-4)^2}$$
$$= \sqrt{(4)^2 + (-2)^2}$$
$$= \sqrt{16 + 4}$$
$$= \sqrt{20}$$

Slope of line segment *AB*:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-2-4}{-6-(-3)}$$
$$= \frac{-6}{-3}$$
$$= 2$$

Slope of line segment *BC*:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-5-(-2)}{0-(-6)}$$
$$= \frac{-3}{6}$$
$$= -\frac{1}{2}$$

Slope of line segment *CD*:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2 - (-5)}{1 - 0}$$
$$= \frac{7}{1}$$
$$= 7$$

Slope of line segment *AD*:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2 - 4}{1 - (-3)}$$
$$= \frac{-2}{4}$$
$$= -\frac{1}{2}$$

While line segments *AB* and *BC* are equal, line segments *CD* and *AD* are not. Line segments *BC* and *AD* are parallel because the line segments have the same slope. Line segment *AB* is perpendicular because its slope is the negative reciprocal of the slope of *BC* and *AD*. However, line segment *CD* is not parallel to line segment *AB* and is not perpendicular to line segments *BC* and *AD*. Quadrilateral *ABCD* is a trapezoid.

15.3 Determining If a Given Point Is on a Circle When the Center Is at the Origin

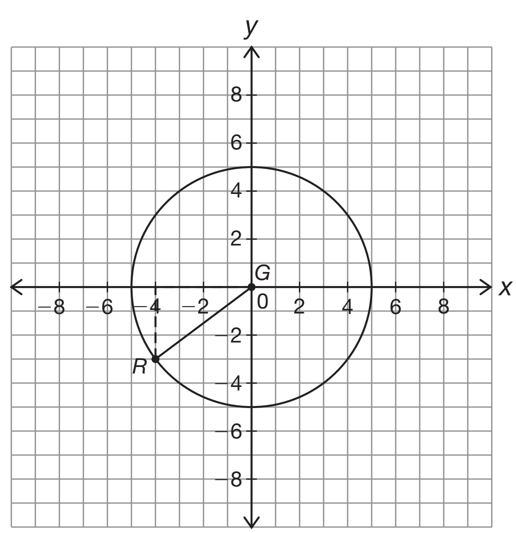
When given a circle with the center at the origin and a given point on a circle the circle can be drawn and the radius determined. To determine if another point lies on the circle, the Pythagorean Theorem or the Distance Formula can be used. If the length of the line segment from the origin to the point is equal to the radius, then the point lies on the circle.

Example

Determine if point $R(-4, -3)$ lies on circle G.

$$GR = \sqrt{(-4 - 0)^2 + (-3 - 0)^2}$$
$$= \sqrt{(-4)^2 + (-3)^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$= 5$$

$$a^2 + b^2 = c^2$$
$$4^2 + 3^2 = c^2$$
$$16 + 9 = c^2$$
$$25 = c^2$$
$$\sqrt{25} = c$$
$$5 = c$$



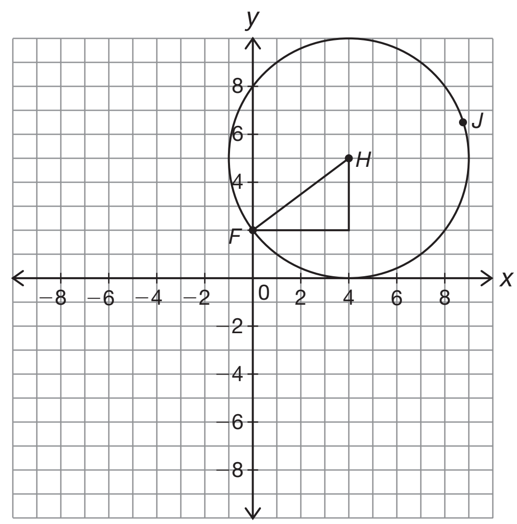
Circle G has a radius of 5 units. Point R lies on circle G because the distance between point G and point R is equal to the radius of circle G.

15.3 Determining If a Given Point Is on a Circle When the Center Is Not at the Origin

When given a circle with the center not at the origin and the location of a point on the circle, it can be determined if a third point lies on the circle. First determine the radius of the circle by using the Pythagorean Theorem. Then use the Distance Formula to determine the distance from the origin to the third point. If the distance is equal to the radius, the point lies on the circle.

Example

Circle *H* has its center at (4, 5) and contains point *F*(0, 2). Determine if point *J*(8.75, 6.5) lies on the circle.



$4^2 + 3^2 = c^2$

$16 + 9 = c^2$

$25 = c^2$

$5 = c$

$HJ = \sqrt{(8.75 - 4)^2 + (6.5 - 5)^2}$

$= \sqrt{(4.75)^2 + (1.5)^2}$

$= \sqrt{22.5625 + 2.25}$

$= \sqrt{24.8125}$

$HJ \approx 4.98$

The distance between point *J* and the origin is not equal to the radius. Point *J* does not lie on the circle.

15.3 Determining If a Given Point Is on a Given Circle after a Transformation

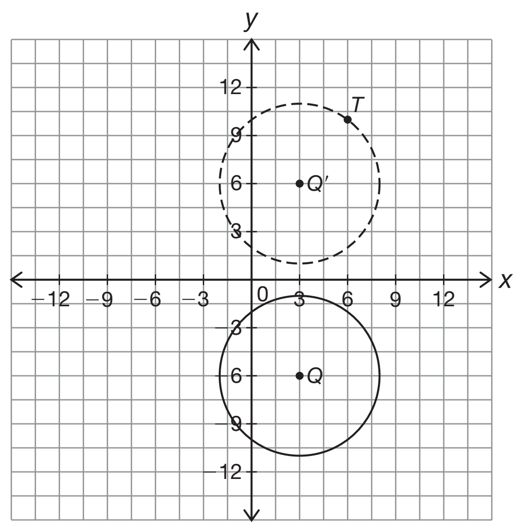
The pre-image of a circle reflected over the x-axis can be used to determine the image's center. The x-coordinate of the center remains the same and the y-coordinate of the image's center is the opposite of the pre-image's center. The radius of the image will be the same as the pre-image because the image remains congruent to the pre-image. The Pythagorean Theorem or the Distance Formula can be used to determine if a point lies on the image.

Example

Circle Q has its center at (3, −6) and has a radius of 5 units. Circle Q is reflected over the x-axis to created Circle Q'. Determine if point T(6, 10) lies on circle Q'.

$Q' = (3, 6)$

$$
\begin{aligned}
Q'T &= \sqrt{(6-3)^2 + (10-6)^2} \\
&= \sqrt{3^2 + 4^2} \\
&= \sqrt{9 + 16} \\
&= \sqrt{25} \\
&= 5
\end{aligned}
$$



Point T(6, 10) lies on circle Q' because the distance of point T from point Q' is equal to the radius of circle Q, which is equal to the radius of its reflection.

15.4 Determining the Coordinates of Points on a Circle Given the Center and Radius

When given a circle with its center at the origin and the radius, points that lie on the circle in each quadrant can be determined using the Pythagorean Theorem. A line segment drawn from the center to any point on the x-axis that is less than the radius represents one side of a right triangle. A vertical line connecting the endpoint of this line segment to the circle represents the other side of a right triangle and is unknown. The distance from the center of the circle to the point on the circle, the radius, represents the hypotenuse.

Example

Circle A is centered at the origin and has a radius of 6 units. Determine a point that lies on the circle in each quadrant.

Point in Quadrant 1:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 4^2 + b^2 &= 6^2 \\ 16 + b^2 &= 36 \\ b^2 &= 20 \\ b &= 2\sqrt{5} \end{aligned}$$

Point in Quadrant 2:

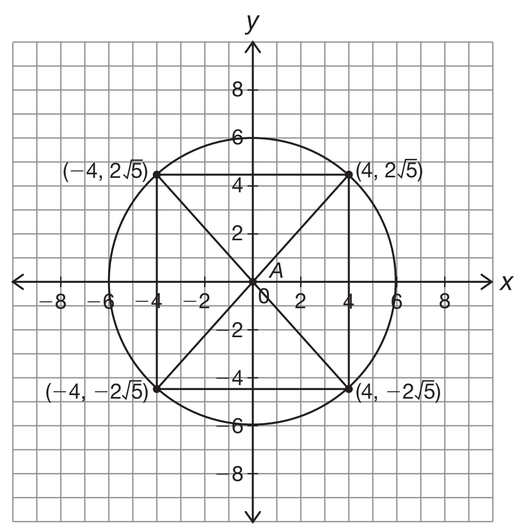
$$\begin{aligned} a^2 + b^2 &= c^2 \\ (-4)^2 + b^2 &= 6^2 \\ 16 + b^2 &= 36 \\ b^2 &= 20 \\ b &= 2\sqrt{5} \end{aligned}$$

Point in Quadrant 3:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (-4)^2 + (-b)^2 &= 6^2 \\ 16 + (-b)^2 &= 36 \\ -b^2 &= 20 \\ b &= -2\sqrt{5} \end{aligned}$$

Point in Quadrant 4:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 4^2 + (-b)^2 &= -6^2 \\ 16 + (-b)^2 &= 36 \\ -b^2 &= 20 \\ b &= -2\sqrt{5} \end{aligned}$$



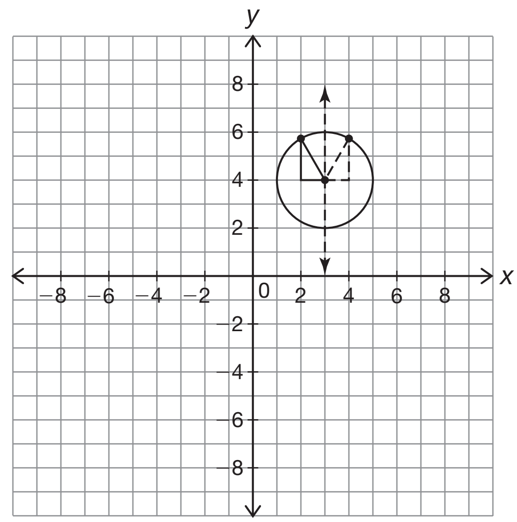
Radius of circle *A*	***x*-Intercepts**	***y*-Intercepts**	**Point in Quad 1**	**Point in Quad 2**	**Point in Quad 3**	**Point in Quad 4**
4	(6, 0) (−6, 0)	(0, 6) (0, −6)	$(4, 2\sqrt{5})$	$(-4, 2\sqrt{5})$	$(-4, -2\sqrt{5})$	$(4, -2\sqrt{5})$

15.4 Determining Points on a Circle that Does Not Have Its Center at the Origin

The Pythagorean Theorem can also be used to determine points on a circle that is not centered on the origin. Because the center is not at the origin, the distance from the circle's center to the origin must be added or subtracted to the coordinate of the new point. The created triangle can then be reflected over the center of the circle to determine the location of a second point on the circle.

Example

Circle M is centered at the point of (3, 4) and has a radius of 2 units. Determine a point that lies on the circle.



Point on circle M:

$a^2 + b^2 = c^2$

$1^2 + b^2 = 2^2$

$1 + b^2 = 4$

$b^2 = 3$

$b = \sqrt{3}$

Add the number of units from the center to the origin to b: $4 + \sqrt{3}$.

A second point on the circle is determined by reflecting the triangle over $x = 3$.

Center	Radius	Points Above and Below Center	Points to the Left and Right of Center	Point on circle M (not on x- or y-axis)	Second point on circle M (result of reflection)
(3, 4)	2 units	(3, 2) (3, 6)	(1, 4) (5, 4)	$(2, 4 + \sqrt{3})$	$(4, 4 + \sqrt{3})$

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