Carnegie Learning Integrated Math I

Student Edition

Volume 2

Carnegie Learning

Carnegie Learning

437 Grant St., Suite 918
Pittsburgh, PA 15219
Phone 412.690.2442
Customer Service Phone 877.401.2527
Fax 412.690.2444

www.carnegielearning.com

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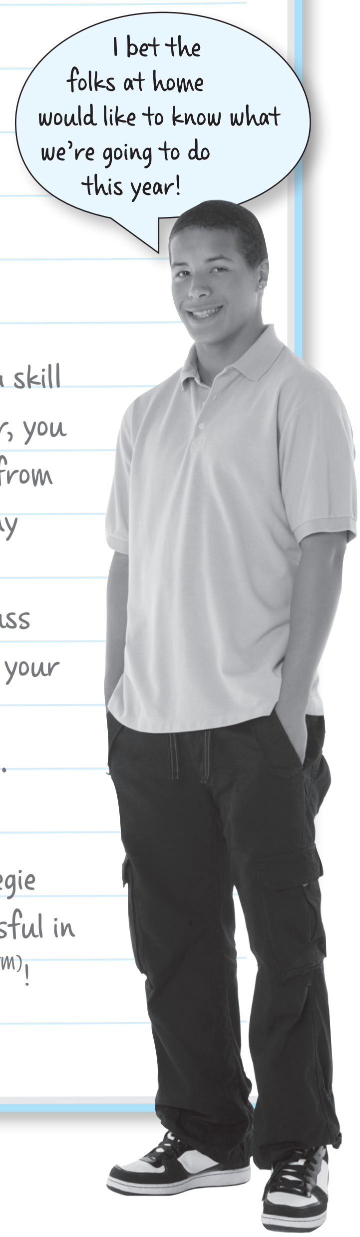
Dear Student,

You are about to begin an exciting endeavor using mathematics! To be successful, you will need the right tools. This book is one of the most important tools you will use this year. Throughout this book there is space for note-taking, sketching, and calculating. You will be given opportunities to think and reason about various mathematical concepts and use tools such as tables, graphs, and graphing calculators.

This year you will face many new challenges both in and outside of the classroom. While some challenges may seem difficult, it is important to remember that effort matters. You must realize that it may take hard work and perseverance to succeed—and your hard work will pay off!

Connections in mathematics are important. Throughout this text, you will build new knowledge based upon your prior knowledge. It is our goal that you see mathematics as relevant because it provides a common and useful language for discussing and solving real-world problems.



Don't worry—you will not be working alone. Working with others is a skill that you will need throughout your life. When you begin your career, you will most likely work with all sorts of people, from shy to outgoing, from leaders to supporters, from innovators to problem solvers—and many more types of people! Throughout this book, you will have many opportunities to work with your classmates. You will be able to discuss your ideas and predictions to different problem situations; present your calculations and solutions to questions; and analyze, critique and suggest, or support your classmates' answers to problem situations.

Today's workplace demands teamwork and self-confidence. At Carnegie Learning, our goal is to provide you with opportunities to be successful in your math course. Enjoy the year and have fun Learning by Doing(TM)!

—The Carnegie Learning Curriculum Development Team

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Carnegie Learning Authoring Team

- Sandy Bartle
  Senior Academic Officer
- Joshua Fisher
  Math Editor
- David "Augie" Rivera
  Math Editor
- David Dengler
  Director, Curriculum Development
- Jen Dilla
  Editorial Assistant
- Lezlee Ross
  Curriculum Developer

Contributing Authors

- Jaclyn Snyder
- Dr. Mary Lou Metz

Vendors

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The Crew

The Crew is here to help you throughout this text. Sometimes they will remind you about things you have already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are members of your group—someone you can rely on!

Teacher aides will guide you along your way. They will help you make connections and remind you to think about the details.

Mathematical Representations

Introduction

During this course, you will solve problems and work with many different representations of mathematical concepts, ideas, and processes to better understand the world. Each lesson will provide you with opportunities to discuss your ideas, work within groups, and share your solutions and methods with your class. These process icons are placed throughout the text.

Discuss to Understand

- Read the problem carefully.
- What is the context of the problem? Do we understand it?
- What is the question that we are being asked? Does it make sense?
- Is this problem similar to some other problem we know?

Think for Yourself

- Do I need any additional information to answer the question?
- Is this problem similar to some other problem that I know?
- How can I represent the problem using a picture, a diagram, symbols, or some other representation?

Work with Your Partner

- How did you do the problem?
- Show me your representation.
- This is the way I thought about the problem—how did you think about it?
- What else do we need to solve the problem?
- Does our reasoning and our answer make sense to each other?
- How will we explain our solution to the class?

Share with the Class

- Here is our solution and the methods we used.
- Are we communicating our strategies clearly?
- We could only get this far with our solution. How can we finish?
- Could we have used a different strategy to solve the problem?

Academic Glossary

Key Terms of the Course

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas. The Graphic Organizers shown display a definition for a key term, related words, sample questions, and examples.

Academic Glossary

Analyze

Definition

To study or look closely for patterns.

Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Related Words

- examine
- evaluate
- determine
- observe
- consider
- investigate
- what do you notice?
- what do you think?
- sort and match
- identify

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?
- What is the question asking me to accomplish?
- What is the context?
- What does the solution mean in terms of this problem situation?

Example

Alan's camping troop hikes down from their campsite at an elevation of 4800 feet to the bottom of the mountain. They hike down at a rate of 20 feet per minute.

1. Write a function, $h(m)$, to show the troop's elevation as a function of time in minutes.

   $h(m) = -20m + 4800$

2. Analyze the function.

   a. Identify the independent and dependent quantities and their units.

   The independent quantity is the number of minutes hiked, and the dependent quantity is the elevation in feet.

   b. Identify the rate of change and explain what it means in terms of this problem situation.

   The rate of change is -20. This represents a decrease of 20 feet every minute.

   c. Identify the *y*-intercept and explain what it means in terms of this problem situation.

   The *y*-intercept is 4800. This shows that the troop started their hike at an elevation of 4800 feet.

   d. What is the *x*-intercept and explain what it means in terms of this problem situation?

   $$0 = -20m + 4800$$
   $$-4800 = -20m$$
   $$\frac{-4800}{-20} = \frac{-20m}{-20m}$$
   $$240 = m$$

   The *x*-intercept is (240, 0). The hikers will be at the bottom of the mountain in 240 minutes, or 4 hours.

Definition

To give details or describe how to determine an answer or solution.

Explaining your reasoning helps justify conclusions.

Related Words

- show your work
- explain your calculation
- justify
- why or why not?

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?
- Did I use complete sentences in my answer?

Explain Your Reasoning

Example

A 747 airliner has an initial climb rate of 1800 feet per minute until it reaches a height of 10,000 feet.

1. Identify the independent and dependent quantities in this problem situation. Explain your reasoning.

   The height of the airplane depends on the time, so height is the dependent quantity and time is the independent quantity.

2. Describe the units of measure for:

   a. the independent quantity (the input values).

   The independent quantity of time is measured in minutes.

   b. the dependent quantity (the output values).

   The dependent quantity of height is measured in feet.

3. Which function family do you think best represents this situation? Explain your reasoning.

   Answers will vary.

   The situation shows a linear function because the rate the plane ascends is constant. So, this situation belongs to the linear function family.

Academic Glossary

Academic Glossary

Definition

To display information in various ways.

Representing mathematics can be done using words, tables, graphs, or symbols.

Related Words

- show
- sketch
- draw
- create
- plot
- graph
- write an equation
- complete the table

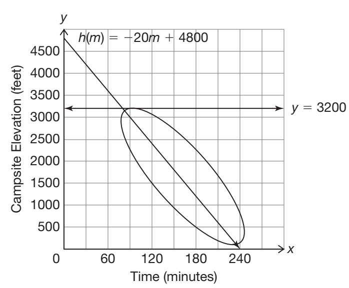
Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?
- What units or labels should I include?
- Are there other ways to model this concept?

Represent

Example

3. Label the function on the coordinate plane.



4. Use the graph to determine how many minutes passed if the troop is below 3200 feet. Draw an oval on the graph to represent this part of the function and write the corresponding inequality statement.

More than 80 minutes has passed if the troop is below 3200 feet.
$m > 80$

5. Write and solve an inequality to verify the solution set you interpreted from the graph.

$$-20m + 4800 < 3200$$
$$-20m + 4800 - 4800 < 3200 - 4800$$
$$-20m < -1600$$
$$\frac{-20m}{-20} < \frac{-1600m}{-20m}$$
$$m > 80$$

Definition

To make an educated guess based on the analysis of given data.

Estimating first helps inform reasoning.

Related Words

- predict
- approximate
- expect
- about how much?

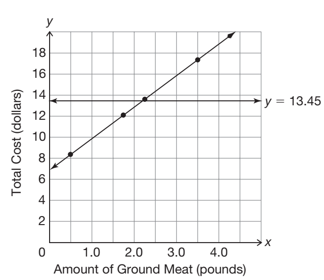
Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?
- What do I know about this problem situation?
- What predictions can I make from this problem situation?

Estimate

Example

3. Use the data from the table to create a graph of the problem situation on the coordinate plane.



4. Consider a total bill of $13.45.

   a. Estimate the amount of ground beef purchased.

   The graph of $y = 13.45$ crosses the original graph at about 2, so I predict that 2 pounds of ground meat were purchased.

   b. Determine the exact amount of ground meat purchased.

   Using the intersection function on my graphing calculator, I determined the exact amount to be 2.2 pounds of ground meat.

Describe

Definition

To represent or give an account of in words.

Describing communicates mathematical ideas to others.

Related Words

- demonstrate
- label
- display
- compare
- define
- determine
- what are the advantages?
- what are the disadvantages?
- what is similar?
- what is different?

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?
- Did I use complete sentences in my answer?
- Did I include appropriate units and labels?
- Will my classmates understand my reasoning?

Example

You just worked with different representations of a linear function.

1. Describe how a linear function is represented:

   a. in a table.

   When the input values in a table are in successive order and the first differences of the output values are constant, the table represents a linear function.

   b. in a graph.

   A linear function is represented in a graph by a straight line.

   c. in an equation.

   A linear function is represented by a function in the form $f(x) = ax + b$.

2. Name some advantages and disadvantages of the graphing method and the algebraic method when determining solutions for linear functions.

   Answers will vary.

   Graphs provide visual representations of functions, and they can provide a wide range of values, depending on the intervals. A disadvantage is that I have to estimate values if points do not fall exactly on grid line intersections. The algebraic method provides an exact solution for every input, but I may be unable to solve more difficult equations correctly.

Academic Glossary

Problem Types You Will See

Worked Example

WHEN YOU SEE A WORKED EXAMPLE

- Take your time to read through it,
- Question your own understanding, and
- Think about the connections between steps.

ASK YOURSELF

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

Consider the sequence shown.

1, 2, 4, 8, . . .

The pattern is to multiply each term by the same number, 2, to determine the next term.

Sequence: $1 \xrightarrow{\text{multiply by } 2} 2 \xrightarrow{\text{multiply by } 2} 4 \xrightarrow{\text{multiply by } 2} 8$, . . .

This sequence is geometric and the common ratio r is 2.

3. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is 3.

a. How would the pattern change?

The sequence would still increase, but the terms would be different.
The sequence would increase more rapidly.

b. Is the sequence still geometric? Explain your reasoning.

Yes. The sequence is still geometric because the ratio between any two consecutive terms is constant.

c. If possible, write the first 5 terms for the new sequence.

1, 3, 9, 27, 81

Thumbs Down

WHEN YOU SEE A THUMBS DOWN ICON

- Take your time to read through the *incorrect* solution.
- Think about what error was made.

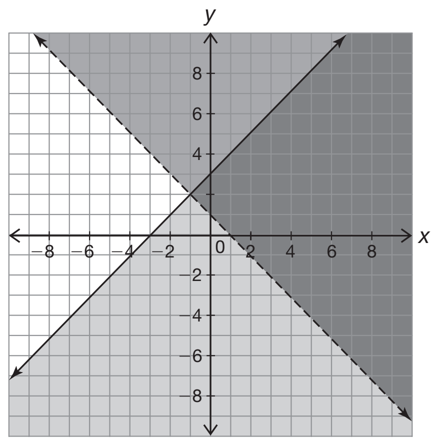
ASK YOURSELF

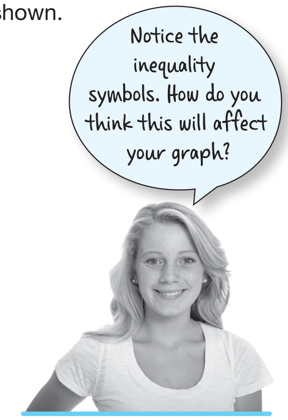
- Where is the error?
- Why is it an error?
- How can I correct it?

5. Analyze the solution set of the system of linear inequalities shown.

$$\begin{cases} x + y > 1 \\ -x + y \leq 3 \end{cases}$$

a. Graph the system of linear inequalities.





c. Alan makes the statement shown.

Alan

The intersection point is always an algebraic solution to a system of inequalities because that is where the two lines meet.

Explain why Alan's statement is incorrect. Use the intersection point of this system to explain your reasoning.

$(-1, 2)$

$-1 + 2 > 1$ $\quad\quad$ $-(-1) + 2 \leq 3$

$1 > 1$ $\quad\quad$ $3 \leq 3$

Alan is incorrect because the intersection point is not always a solution to the system of linear inequalities. The intersection point for this system only works for one of the inequalities, not both which means it is not a solution. If the inequality symbols are not both "or equal to" then the intersection point is not a solution.

Problem Types

Thumbs Up

WHEN YOU SEE A THUMBS UP ICON

- Take your time to read through the *correct* solution.
- Think about the connections between steps.

ASK YOURSELF

- Why is this method correct?
- Have I used this method before?

8. Pat and George each wrote a function to represent the number of rice grains for any square number using different methods.

Pat

I compared the exponents of the power to the square number in the table. Each exponent is 1 less than the square number.

$f(s) = 2^{s-1}$

George

I know this is an exponential function with a common base of 2. If I extend the pattern back on the graph I get the y-intercept of $\left(0, \frac{1}{2}\right)$, so $a = \frac{1}{2}$.

$f(s) = \frac{1}{2}(2)^s$

Use properties of exponents to verify that 2^{s-1} and $\frac{1}{2}(2)^s$ are equivalent.

$$
\begin{aligned}
2^{s-1} &= (2^s)(2^{-1}) \\
&= (2^s)\left(\frac{1}{2}\right) \\
&= \left(\frac{1}{2}\right)(2^s)
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{2}(2)^s &= (2)^{-1}(2)^s \\
&= 2^{-1+s} \\
&= 2^{s-1}
\end{aligned}
$$

Problem Types

Who's Correct?

WHEN YOU SEE A WHO'S CORRECT? ICON

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine which solution is correct and which is not correct.

ASK YOURSELF

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

8. Carlos and Mikala do not like working with fractions. They rewrite their equation so that it does not have fractions. Their work is shown.

Carlos

$F = \frac{9}{5}C + 32$

$(5)F = 5\left(\frac{9}{5}C + 32\right)$

$5F = 9C + 160$

$5F - 9C = 160$

Mikala

$C = \frac{5}{9}(F - 32)$

$(9)C = (9)\frac{5}{9}(F - 32)$

$9C = 5(F - 32)$

$9C = 5F - 160$

$9C - 5F = -160$

Carlos and Mikala got two different equations. Who is correct?

Both Carlos and Mikala are correct. If they divide either equation by −1 they will get the other equation.

The Standards for Mathematical Practice

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker.

Make sense of problems and persevere in solving them.

I can:

- explain what a problem "means" in my own words.
- analyze and organize information.
- keep track of my plan and change it if necessary
- always ask myself, "does this make sense?"

Attend to precision.

I can:

- calculate accurately and efficiently.
- use clear definitions when I talk with my classmates, my teacher, and others.
- specify units of measure and label diagrams and other figures appropriately to clarify the meaning of different representations.

Reasoning and Explaining

Reason abstractly and quantitatively.

I can:

- create an understandable representation of a problem situation.
- consider the units of measure involved in a problem.
- understand and use properties of operations.

Construct viable arguments and critique the reasoning of others.

I can:

- use definitions and previously established results in constructing arguments.
- communicate and defend my own mathematical reasoning using examples, drawings, or diagrams.
- distinguish correct reasoning from reasoning that is flawed.
- listen to or read the conclusions of others and decide whether they make sense.
- ask useful questions in an attempt to understand other ideas and conclusions.

Modeling and Using Tools

Model with mathematics.

I can:

- identify important relationships in a problem situation and represent them using tools such as, diagrams, tables, graphs, and formulas.
- apply mathematics to solve problems that occur in everyday life.
- interpret mathematical results in the contexts of a variety of problem situations.
- reflect on whether my results make sense, improving the model I used if it is not appropriate for the situation.

Use appropriate tools strategically.

I can:

- use a variety of different tools that I have to solve problems.
- use a graphing calculator to explore mathematical concepts.
- recognize when a tool that I have to solve problems might be helpful and also when it has limitations.

Habits of Mind

Seeing Structure and Generalizing

Look for and make use of structure.

I can:

- look closely to see a pattern or a structure in a mathematical argument.
- can see complicated things as single objects or as being composed of several objects.
- can step back for an overview and can shift my perspective.

Look for and express regularity in repeated reasoning.

I can:

- notice if calculations are repeated.
- look for general methods and more efficient methods to solve problems.
- evaluate the reasonableness of intermediate results.
- make generalizations based on results.

Each lesson provides opportunities for you to think, reason, and communicate mathematical understanding. Here are a few examples of how you will develop expertise using the Standards for Mathematical Practice throughout this text.

PROBLEM 1 As We Make Our Final Descent

At 36,000 feet, the crew aboard the 747 airplane begins making preparations to land. The plane descends at a rate of 1500 feet per minute until it lands.

Reason abstractly and quantitatively. You will move from a real-life context to the mathematics and back to the context throughout problems.

1. Compare this problem situation to the problem situation in Lesson 2.1, *The Plane!* How are the situations the same? How are they different?

2. Complete the table to represent this problem situation.

	Independent Quantity	Dependent Quantity
Quantity		
Units		
	0	
	2	
	4	
	6	
		18,000
		6000
Expression	t	

Model with mathematics. You will identify relationships and represent them using diagrams, tables, graphs, and formulas.

Look for and make use of structure. You will look for patterns in your calculations and use those to write formal expressions and equations.

Habits of Mind

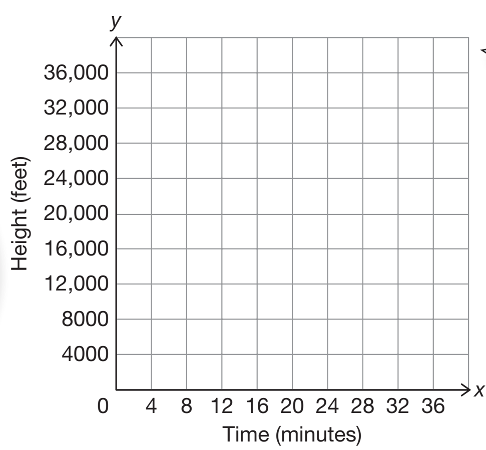
4. Complete the table shown. First, determine the unit of measure for each expression. Then, describe the contextual meaning of each part of the function. Finally, choose a term from the word box to describe the mathematical meaning of each part of the function.

input value	output value	rate of change
y-intercept	x-intercept	

		Description	
Expression	**Units**	**Contextual Meaning**	**Mathematical Meaning**
t			
-1500			
$-1500t$			
36,000			
$-1500t + 36{,}000$			

Attend to precision.
You will specify units of measure to clarify meaning.

5. Graph $g(t)$ on the coordinate plane shown.



Use appropriate tools strategically.
You will use multiple representations throughout the text.

Construct viable arguments and critique the reasoning of others.
You will share your answers with your classmates and listen to their responses to decide whether they make sense.